# Exam 2 Practice D 25X

## Problem 1

A delivery driver has 10 packages (6 round, 4 rectangles) to deliver.

i How many different orderings can they deliver the packages in?

ii Suppose they were lazy and chose any 4 packages to deliver, how many ways can they choose only 4 packages?

iii Suppose they were lazy and chose 6 packages to leave in the truck while they delivered the rest, how many ways can they choose 6 packages to leave in the truck? How does this relate to the previous question, does it feel eerily familiar?

iv How many different ways can they choose to 3 round and 3 rectangular packages to deliver?

v Every delivery, the driver enters one of three notes in his log:

- ”I delivered the package (no signature required)”
- ”I delivered the package and got a signature”
- ”I did not deliver the package, but left a note because nobody is home to sign for the package”

Assume that each of the round packages requires a signature while the rectangular packages do not. At the end of the day, the drivers log might look like this:

- ”I left a note because person isn’t home to sign for it”
- ”I delivered the package and got a signature”
- ”I delivered the package and did not get a signature”
- ”I delivered the package and got a signature”
- ”I delivered the package and did not get a signature”
- ”I delivered the package and did not get a signature”
- ”I delivered the package and did not get a signature”

- "I left a note because person isn't home to sign for it"
- "I delivered the package and got a signature"
- "I delivered the package and got a signature"

How many unique logs can the driver have at the end of the day? Note that the log does not include specific information about which package a log message refers to.

**Problem 2**

Given a standard 52-card deck of playing cards (half of cards are red and half are black) one draws 5 cards without replacement. What is the probability that the first three cards drawn are red, and the last two cards drawn are black? (Notice: this event does not include drawing two black cards before then three red cards).

Be sure to show and justify and clearly document your work as well as computing a final probability value.

**Problem 3**

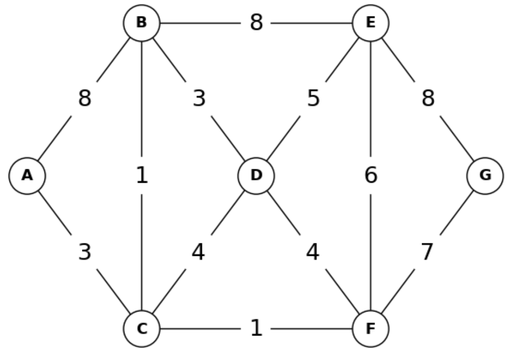
A basketball player makes shots according to the following table:

| | **Distance to net** | **Points earned (if made)** | **Prob shot made (%)** |
|---|---|---|---|
| **P** | In Paint | 2 | 45 |
| **J** | Mid Range Jump | 2 | 40 |
| **T** | 3 pt | 3 | 35 |

Where we use random variables $P, J$ or $T$ to indicate the number of points earned when a player takes each of the corresponding shots.

- i Compute the expected value and variance of: $P$
- ii Compute the expected value and variance of: $J$
- iii Compute the expected value and variance of: $T$
- iv If your team was down by 1 point with time for 1 more shot, which shot should this player prefer to maximize their chance of winning?
- v In the early game, which shot should this player prefer?

**Problem 4**




Using Dijkstra's algorithm, find the shortest path from node A to G. Please provide a table which shows the path weight and predecessor from A to every node, labelling the visited node at each step. an example solution is given here.

**Problem 5 number of trailing 0 bits**

Suppose we write the number 10! (ten factorial) in binary. How many consecutive zeros will occur on its right-hand-side?

**Problem 6 Recurrences, Induction, Big-O**

1. Circle the recursive function that grows the fastest (consider using the back of the exam for scratch if you need it):

   $T(n) = 2T(n/2) + n$ $\quad\quad$ $T(n) = 2T(n-1) + 1$ $\quad\quad$ $T(n) = T(n/2) + 1$

2. Prove by induction: any amount of postage $n \geq 8$ can be achieved using 3 and 4 cent stamps.

3. For each pair of functions f(n) and g(n), put a check mark into each box that corresponds to a relation that holds between the functions.

| $f(n)$ | $g(n)$ | $f(n) = O(g(n))$ | $f(n) = \Theta(g(n))$ | $f(n) = \Omega(g(n))$ |
|---|---|---|---|---|
| $5n$ | $100n$ | ✓ | ✓ | ✓ |
| $\log_2 n$ | $2^n$ | ✓ | | |
| $n^2$ | $n$ | | | ✓ |
| $n \log_2 n$ | $n^2$ | ✓ | | |
| $\log_2 n$ | $\log_3 n$ | ✓ | ✓ | ✓ |