Exam 2 Practice B 25X

Problem 1

i How many ways can one select 6 different colors, from among 15 possible, to put on their palette. Assume that the order of colors on the palette does not matter.

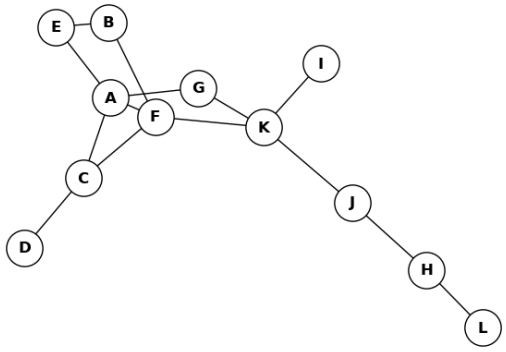
ii Repeat the problem above, but now assume that the order of colors on the palette yields meaningfully different outcomes.

iii How many colors can be made by mixing up to 15 colors in equal proportions? For example, if red and blue are among the original 15 colors then mixing them in equal proportions makes purple. Assume that no color could be made by mixing two different subset of the original 15 colors.

iv A high school has 3 art rooms. The supply closet has 10 tubes of red paint, 7 tubes of blue paint and 5 tubes of green. How many ways can the tubes of paint be distributed among the rooms? (For example:

- room1 gets 4 red, 7 blue and 1 green
- room2 gets 3 red, 0 blue and 1 green
- room3 gets 3 red, 0 blue and 3 green

Problem 2



Wherever possible below, select the node which is earlier in the alphabet first (e.g. prefer visiting node A first over node B, when the search allows you to visit either).

i Starting at L, find the Breadth-First-Search (BFS) ordering of nodes in the graph above.

ii Starting at D, find the Breadth-First-Search (BFS) ordering of nodes in the graph above.

iii Starting at E, find the Breadth-First-Search (BFS) ordering of nodes in the graph above.

iv Starting at L, find the Depth-First-Search (DFS) ordering of nodes in the graph above.

v Starting at D, find the Depth-First-Search (DFS) ordering of nodes in the graph above.

vi Starting at E, find the Depth-First-Search (DFS) ordering of nodes in the graph above.

Problem 3

Tell whether each of the following statements is true or false. No explanation is needed.

i $n^2 + 3 = O(n^2)$

ii $19n = O(n)$

iii $\log_2 n = O(\log_3 n)$

iv $123 = O(n!)$

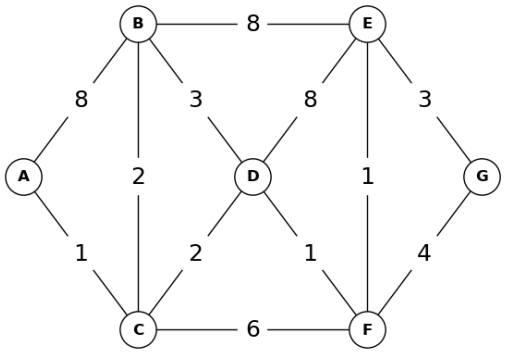
v $n^2 = \Omega(n^2 + 4)$

vi $3 \log_2 n = \Theta(n \log_3 n)$

vii if $f(n) = \Theta(g(n))$ then $f(n) = O(g(n))$

viii For all function f, g, if $f(n) = \Omega(g(n))$ then $f(n) = O(g(n))$

Problem 4



Using Dijkstra's algorithm, find the shortest path from node A to G. Please provide a table which shows the path weight and predecessor from A to every node, labelling the visited node at each step. an example solution is given here.

Problem 5 Probabilities Quiz

i. How many "words" of length 2 through 7 can be formed from a Scrabble rack of 7 distinct letters (assuming every such arrangement is permissible).

ii. How many four-digit numbers are there formed from the digits $1, 2, 3, 4$ that are divisible by 4?

iii. OLD TOPIC REFRESHER QUESTION: Prove that at a party with more than one person present, there is some pair of people who know the same number of other people at the party.

iv. What is the probability that if a die is rolled five times, only two different values appear?

v. Which is more likely, rolling an 8 when two dice are rolled, or rolling an 8 when three dice are rolled?

vi. How many ways are there for 10 people to have 5 simultaneous phone conversations?

Problem 6 ★ Big power mod 10
What is the last digit of 7^{2222} in base 10?