CS1800
Discrete Structures
Fall 2019

Lecture 20
11/15/19

Last time

- Recurrences
- Growth of functions

Today

- Order notation

Next time

- Graphs

Order notation: $o \quad O \quad \Theta \quad \Omega \quad \omega$

$\text{"}<\text{"} \quad \text{"}\leq\text{"} \quad \text{"}=\text{"} \quad \text{"}\geq\text{"} \quad \text{"}>\text{"}$

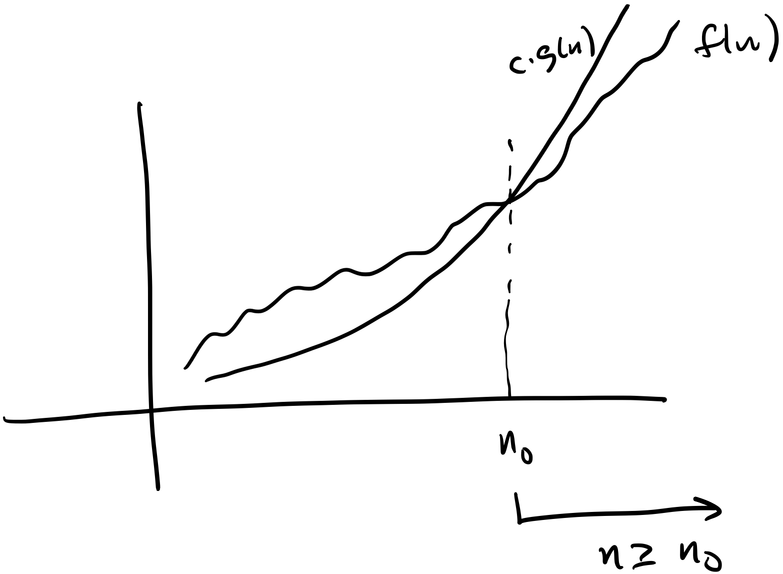
"Big-oh" $f(n) = O(g(n))$ if $\exists$ constants $c, n_0 > 0$ s.t.

$$0 \leq \underline{f(n) \leq c \cdot g(n)} \quad \forall \quad \underline{n \geq n_0}$$

$3n\lg n - 4n + 6 = O(n \lg n)$

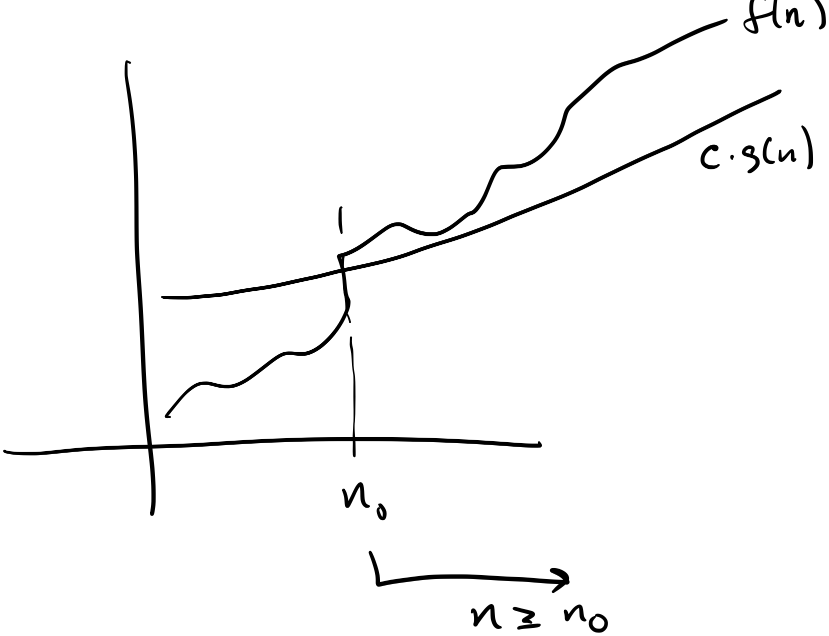
$f(n) = 3n\lg n - 4n + 6$

$g(n) = n \lg n$



"Big Omega" $f(n) = \Omega(g(n))$ if $\exists$ constants $c, n_0 > 0$ s.t.

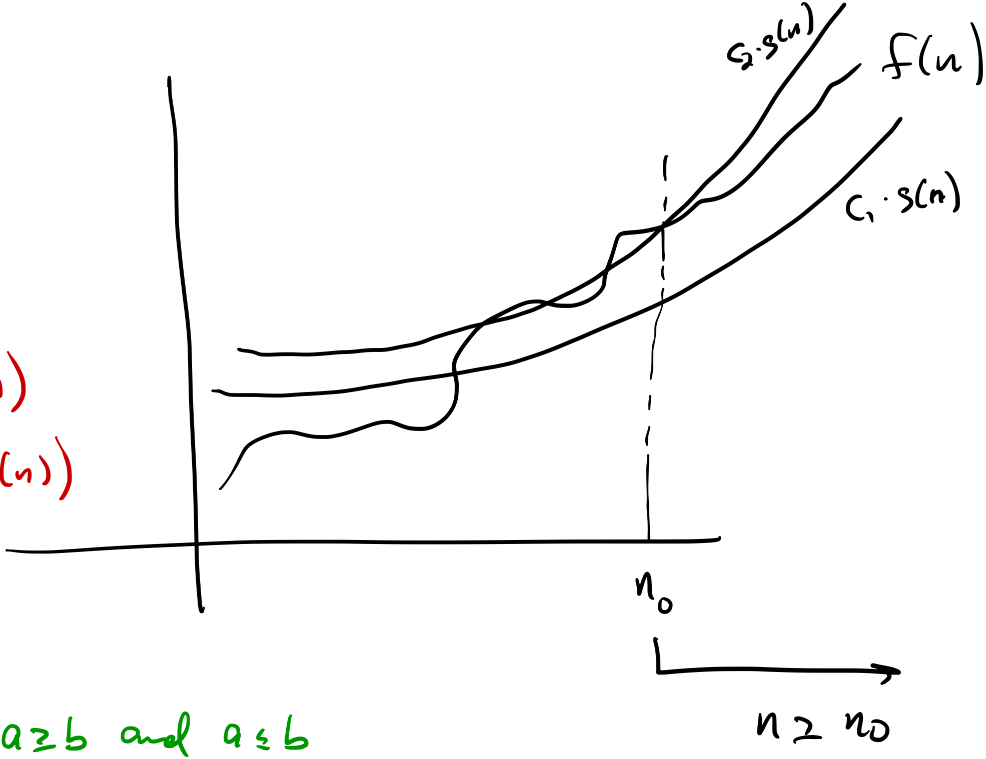
$$0 \le c \cdot g(n) \le f(n) \quad \forall\, n \ge n_0$$



"theta" $f(n) = \Theta(g(n))$ if $\exists$ constants $c_1, c_2, n_0 > 0$

s.t.

$$0 \le c_1 \cdot g(n) \le f(n) \le c_2 \cdot g(n) \quad \forall n \ge n_0$$



$f(n) = \Theta(g(n))$

iff

$f(n) = O(g(n))$

$f(n) = \Omega(g(n))$

analogous to

$a = b$

iff $a \ge b$ and $a \le b$

E.g. $f(n) = \frac{1}{2}n^2 - 2n$

Claim: $f(n) = O(n^2)$

Pf: Must find $c, n_0 > 0$

s.t. $\frac{1}{2}n^2 - 2n \le c \cdot n^2 \quad \forall n \ge n_0$

$c = \frac{1}{2}$ should work

- pick $c = \frac{1}{2}$
- when is $\cancel{\frac{1}{2}n^2} - 2n \le \cancel{\frac{1}{2} \cdot n^2}$

$\iff 0 \le 2n$

$\iff n \ge 0$

pick $n_0 = 1$

Claim: $f(n) = \Omega(n^2)$

pf: Must find constants $c, n_0 \ge 0$ s.t.

$\frac{1}{2}n^2 - 2n \ge c \cdot n^2 \quad \forall n \ge n_0$

- try $c = \frac{1}{4}$
- when is

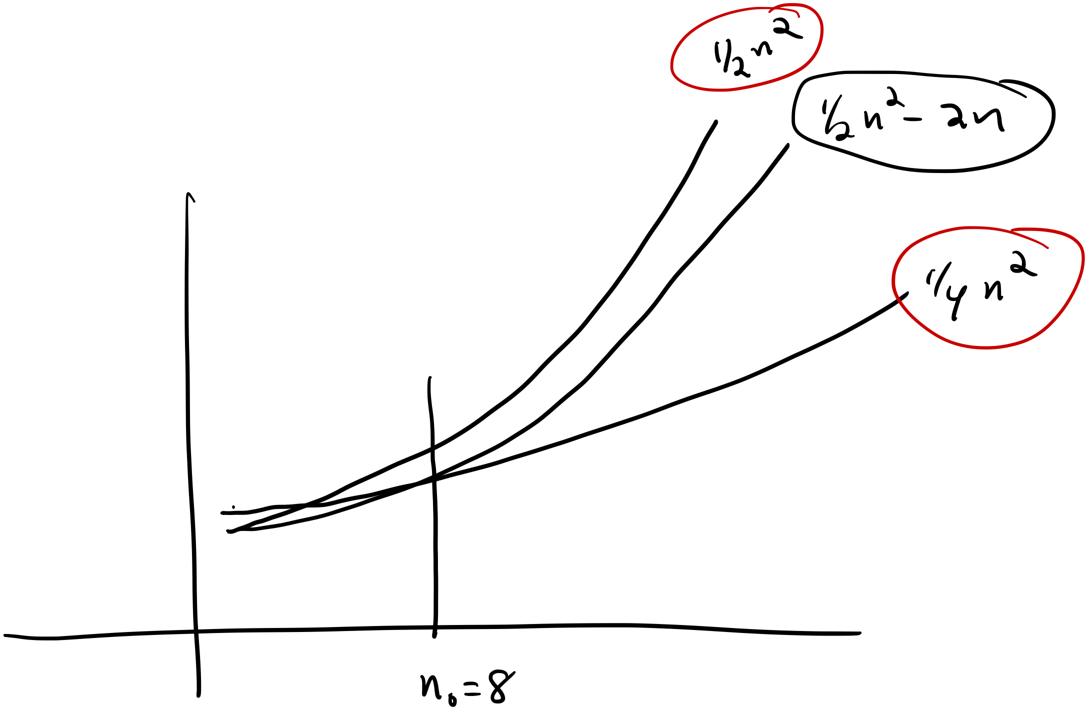
$\frac{1}{2}n^2 - 2n \ge \frac{1}{4}n^2$?

$\iff \frac{1}{4}n^2 \ge 2n$

$\iff \frac{1}{4}n \ge 2$

$\iff n \ge 8$

$c = \frac{1}{4} \quad n_0 = 8$ ✓


$1/2\, n^2$
$1/2\, n^2 - 2n$
$1/4\, n^2$
$n_0 = 8$

E.g. $f(n) = \frac{1}{2}n^2 + 2n$ Show: $f(n) = O(n^2)$

$f(n) \in \underline{O(n^2)}$

set of all "n^2" functions.

- need constants c, n_0 s.t.

$$\frac{1}{2}n^2 + 2n \le c \cdot n^2 \quad \forall n \ge n_0$$

- try $c = 1$

- need $\frac{1}{2}n^2 + 2n \le 1 \cdot n^2$

$$\Leftrightarrow \quad 2n \le \frac{1}{2}n^2$$

$$\Leftrightarrow \quad 2 \le \frac{1}{2}n$$

$$\Leftrightarrow \quad 4 \le n$$

pick $n_0 = 4$

Order notation: $o \quad O \quad \Theta \quad \Omega \quad \omega$

"$<$" "$\le$" "$=$" "$\ge$" "$>$"

"little-oh" o "$<$"

$f(n) = o(g(n))$ if

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} = 0$$

e.g. $f(n) = \ln n$

$g(n) = n^2$

$\ln(n) = o(n^2)$

$$\lim_{n\to\infty} \frac{\ln n}{n^2} = \lim_{n\to\infty} \frac{1/n}{2n}$$

$$= \lim_{n\to\infty} \frac{1}{2n^2} = 0$$

"little-omega" ω "$>$"

$f(n) = \omega(g(n))$ if

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} = \infty$$

$n^2 = \omega(n)$

$$\lim_{n\to\infty} \frac{n^2}{n} = \lim_{n\to\infty} n = \infty$$

$a > b \Rightarrow a \ge b$

$o \Rightarrow O$ $\quad \ln(n) = o(n^2) \Rightarrow$

$\omega \Rightarrow \Omega$ $\quad \ln(n) = O(n^2)$