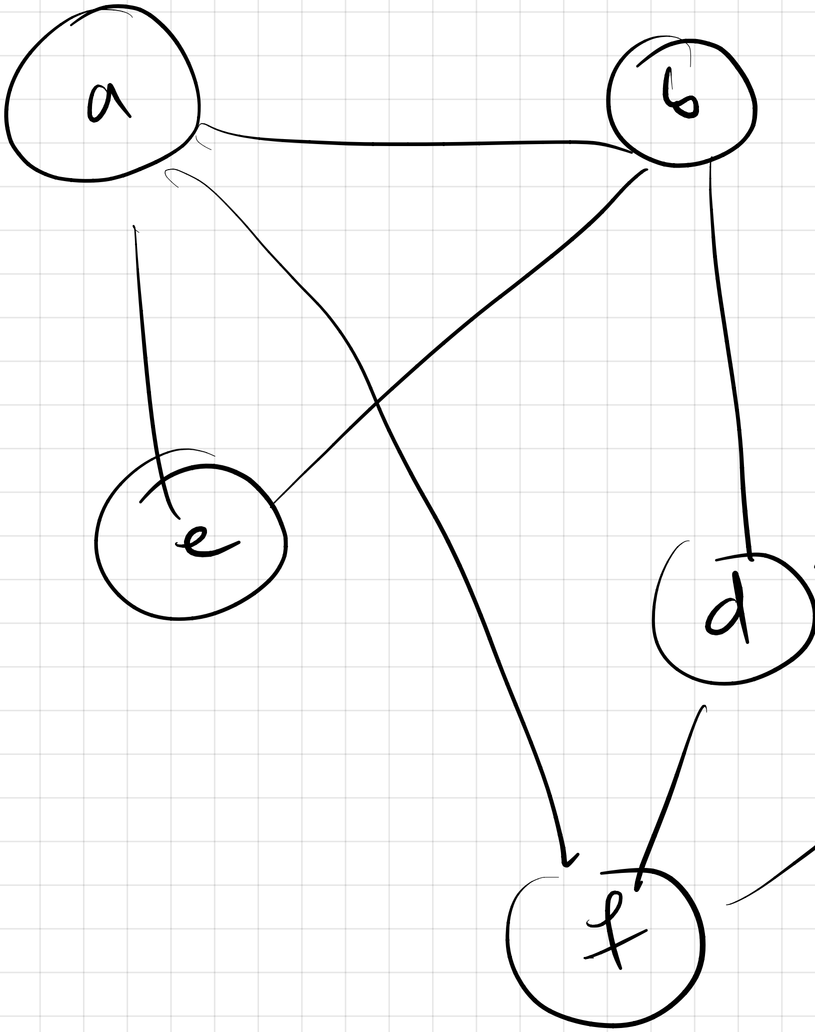
Graphs: vertices / nodes / circles
edges / connections / lines / pairs-of-vertic



V = {a, b, c, ... f}

E = set of edges

informal

{ ab, ae, bd,
be, dc, df,
cf }

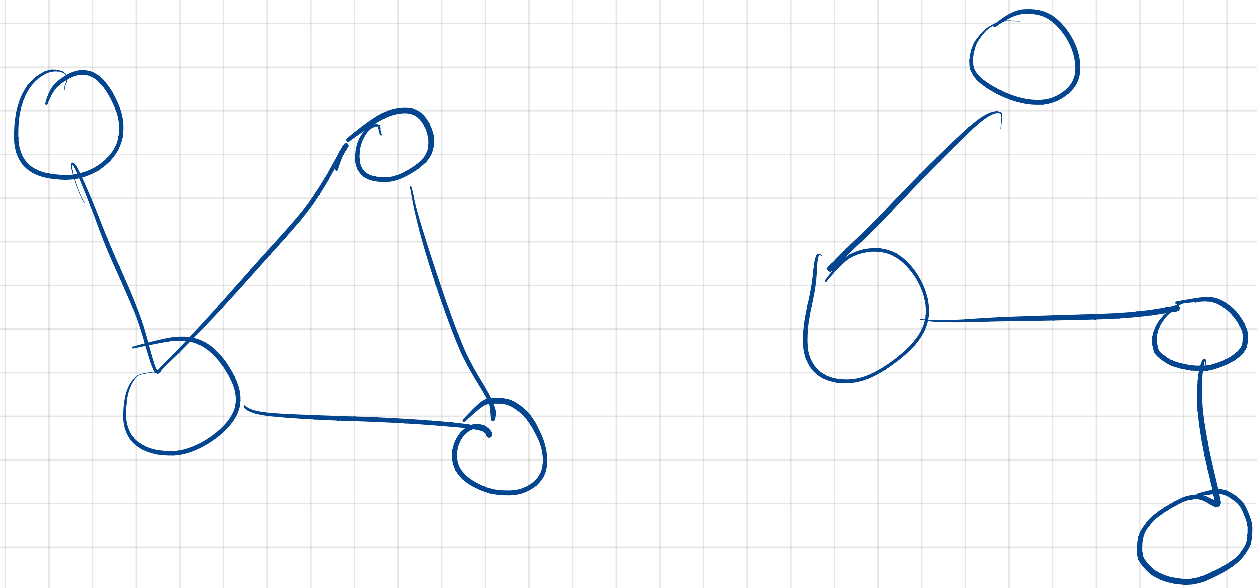
= { (a,b), (a,c),
(b,d) ... }

formal

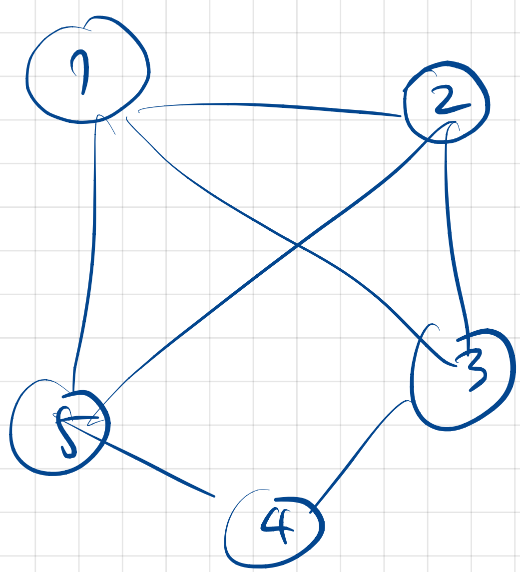
edge a→b, a–b

path: a–b–d–f : a⇝f

Connected : "path" from any vertex u ⇝ vertex v disconnected: 2 connected components



Subgraph: $V' \subset V$ and all corresponding edges



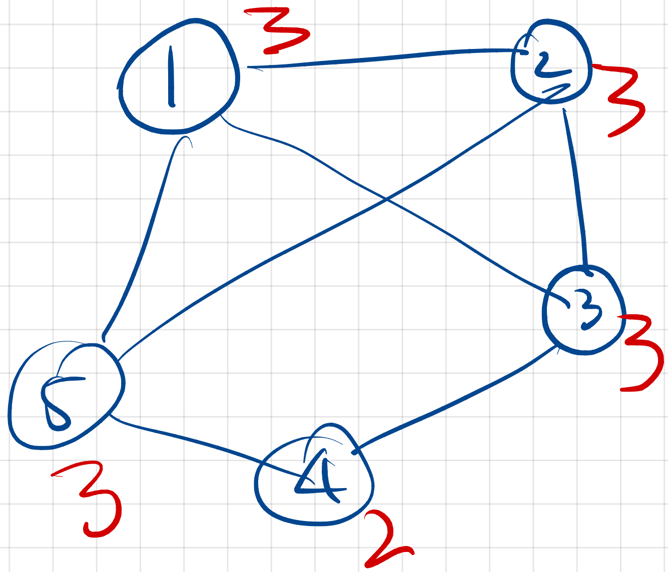
$V = \{1, 2, 3, 4, 5\}$

Subgraph: $V' = \{1, 4, 5\}$

edges $E' = \{15, 45\}$

degree (vertex) deg(a) d(a) deg = red

= # edges incident at that vertex



$\deg(1) = 3$

$\deg(4) = 2$

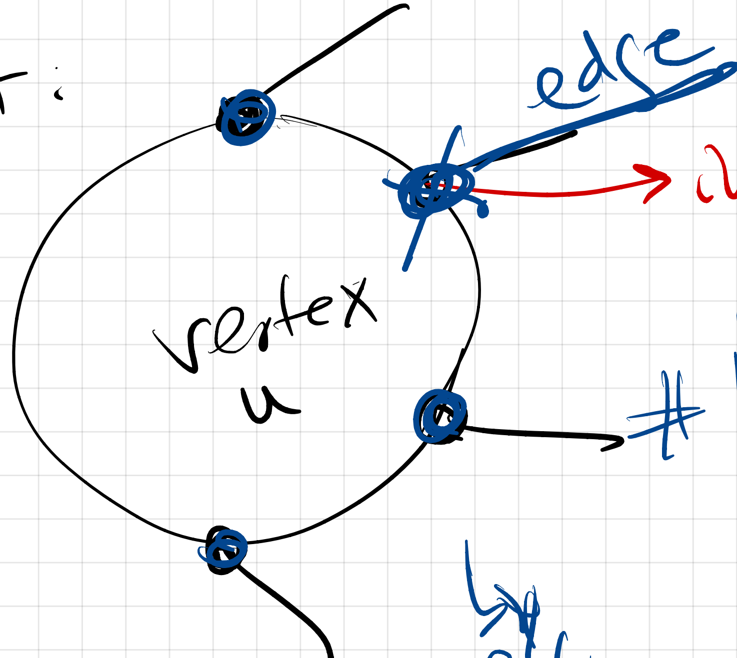
Theorem (hand-shake lemma):

Graph $G = (V, E) = ($vertex set, edge set$)$

$$\sum_{u \in V} \deg(u) = 2|E|$$

sum of vertex deg = twice # of edges

proof:



vertex → (u, edge) ← edge

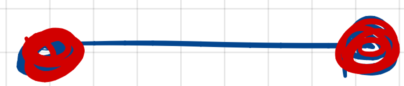
incident point (u, edge)

count # | incident points in two ways:

by vertices	by edges

$$\sum_{u} \deg(u) = \sum_{e \in E} 2$$

deg(u) = # incident in u

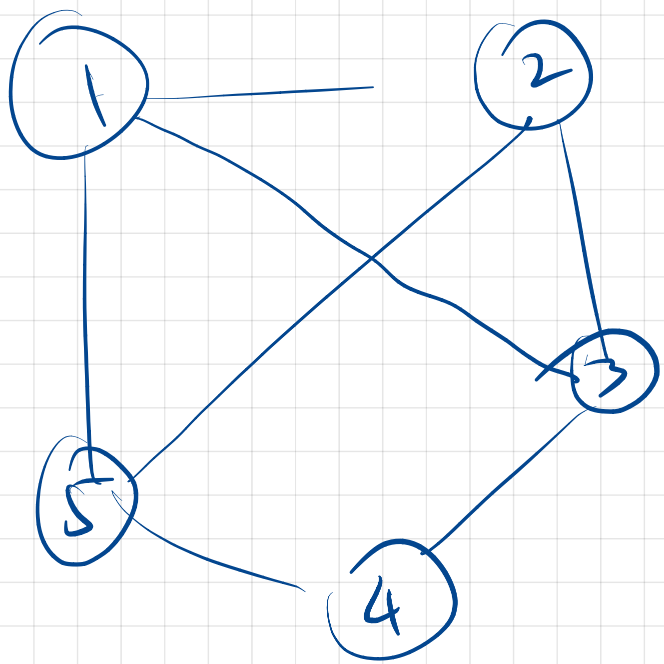


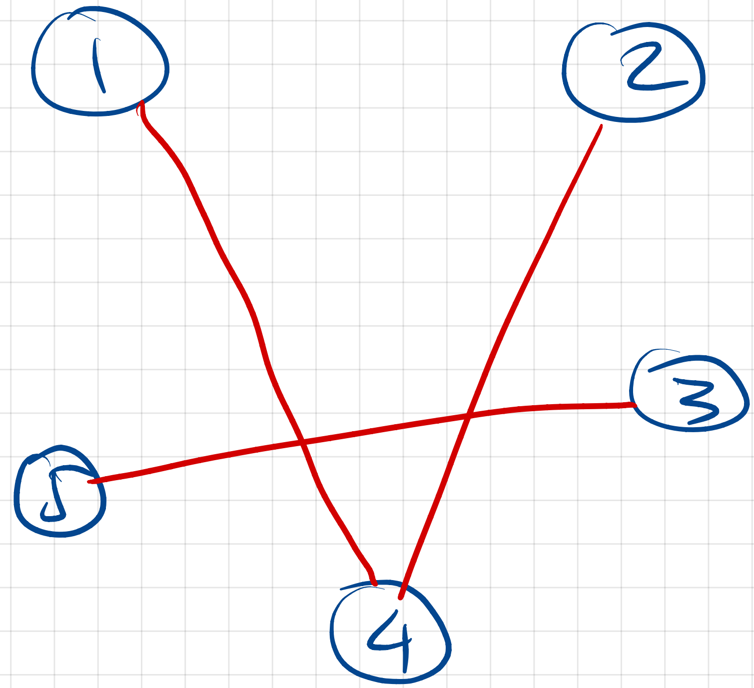
Corolary: # vertices with odd degree = even

deg(u) = odd = 2k+1

Complement(G) = $\overline{G}$ = (V , $\overline{E}$ = missing edges in E)

same





{3,4} clique

Δ = clique of 3 : 125
123

max clique size = 3

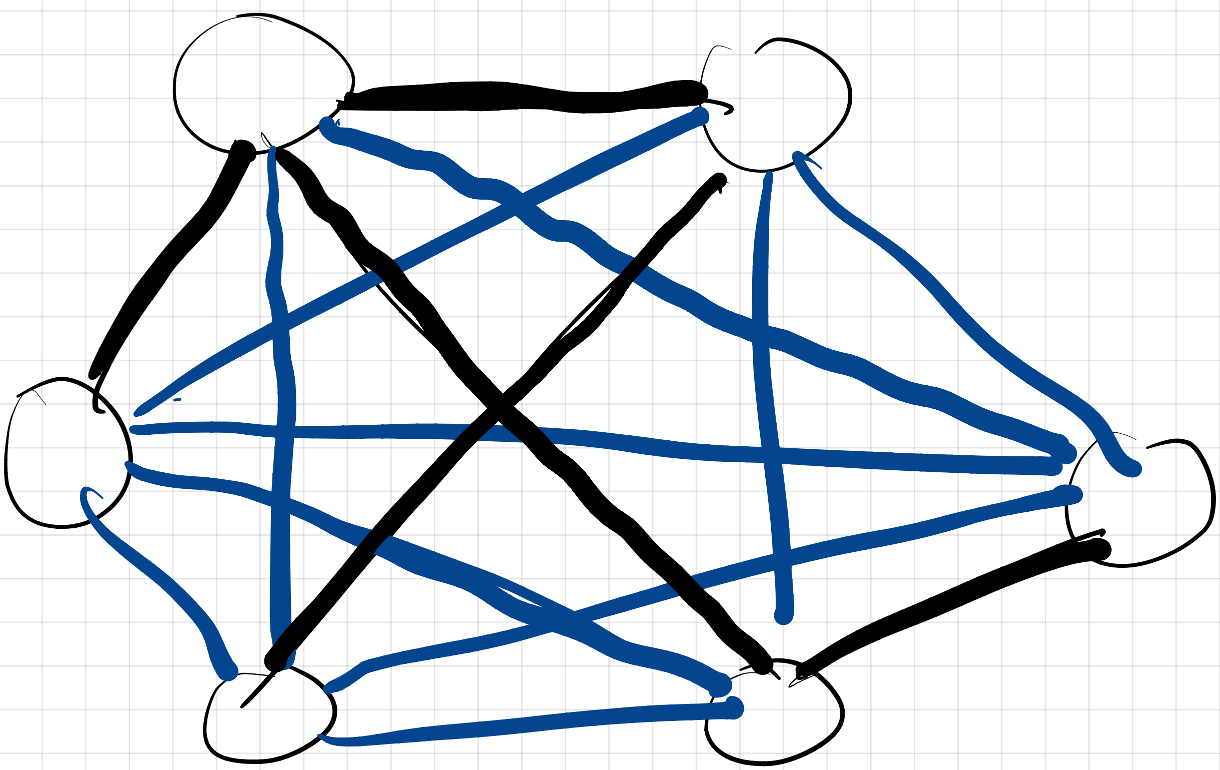
$\overline{E} = \{14, 24, 35\}$

$E \cup \overline{E}$ = all edges (pairs)

<u>clique</u> : subset (or whole set) of vertices with all edges present

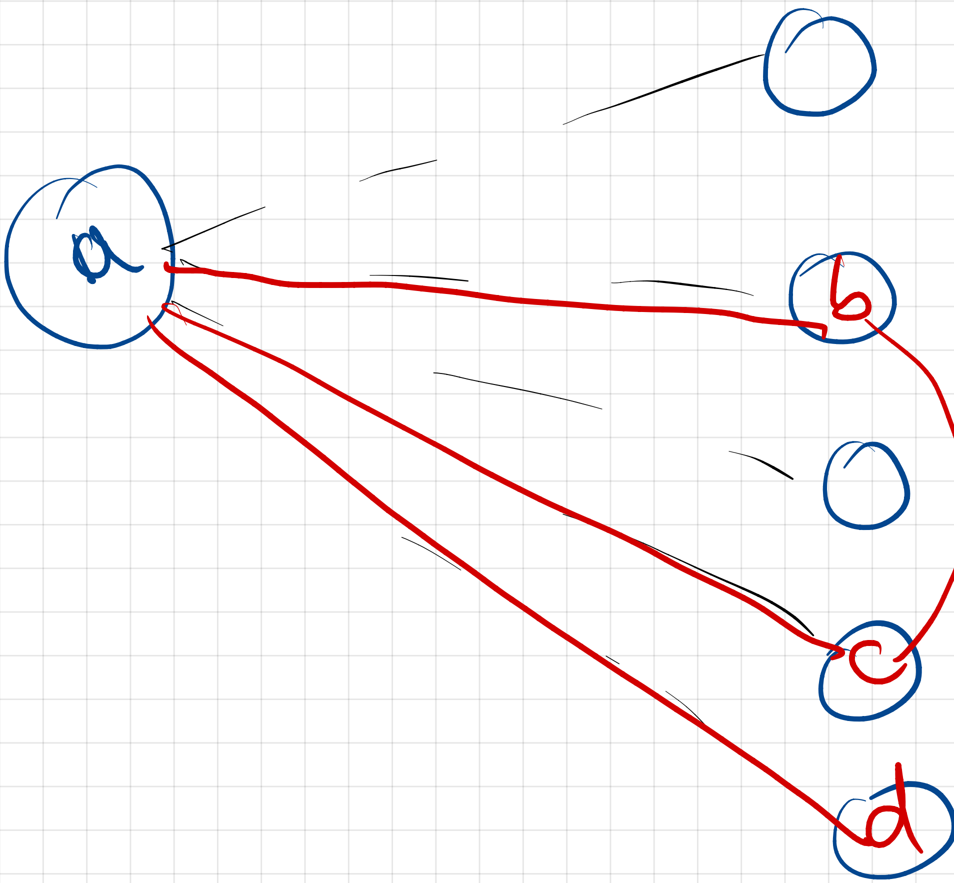
Pb ($*$) $G = (V, E)$ $\quad$ $\overline{G} = (V, \overline{E})$ $\quad$ $|V| = 6$

prove that one of G or $\overline{G}$ has a $|\text{clique}| = 3$ (a triangle)



Proof $a \in V$

look at all possible $\boxed{a\text{-edges } (5)}$



- some are in G
- the other are in $\bar{G}$

PHP $\Rightarrow$ one of the $G/\bar{G}$ has 3 edges ab, ac, ad

"red" graph is either $G/\bar{G}$

b, c, d:

- either they have an edge in same "red" graph

say $bc \Rightarrow \Delta abc$

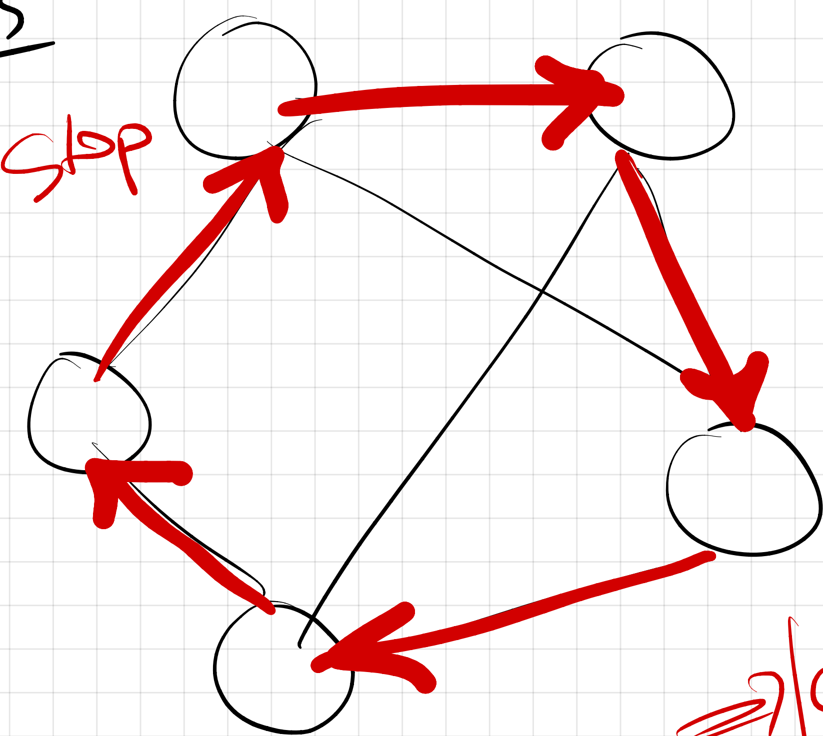
- or no red edge between (b, c, d)

b c d

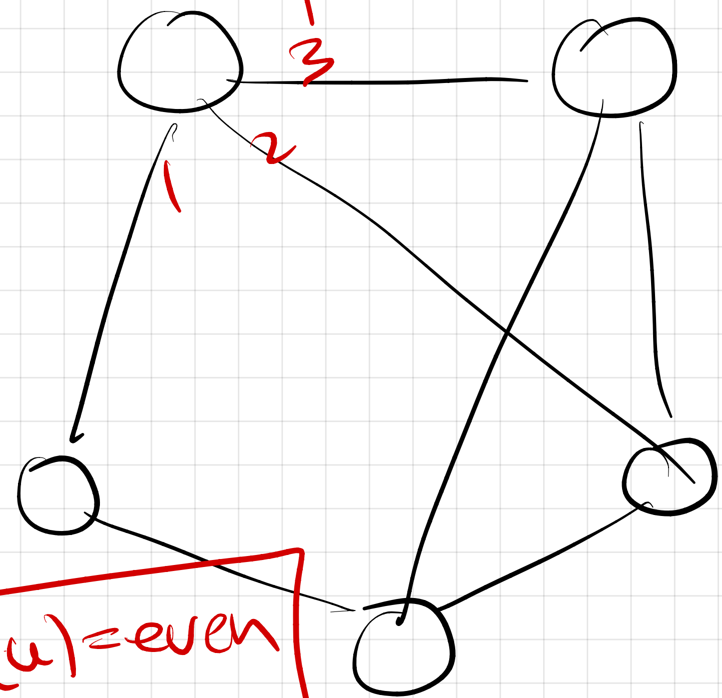
$\Rightarrow$ all 3 edges bc, cd, bd must be in red $\Delta\, bcd$

Tours: path that ends where it started = cycle

cycles



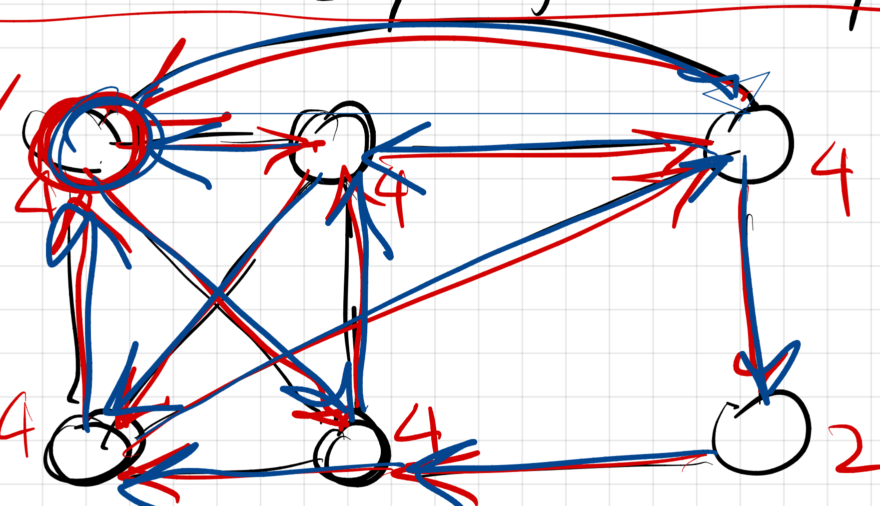
ps = odd degree



$\Rightarrow$ $\boxed{deg(u) = even}$

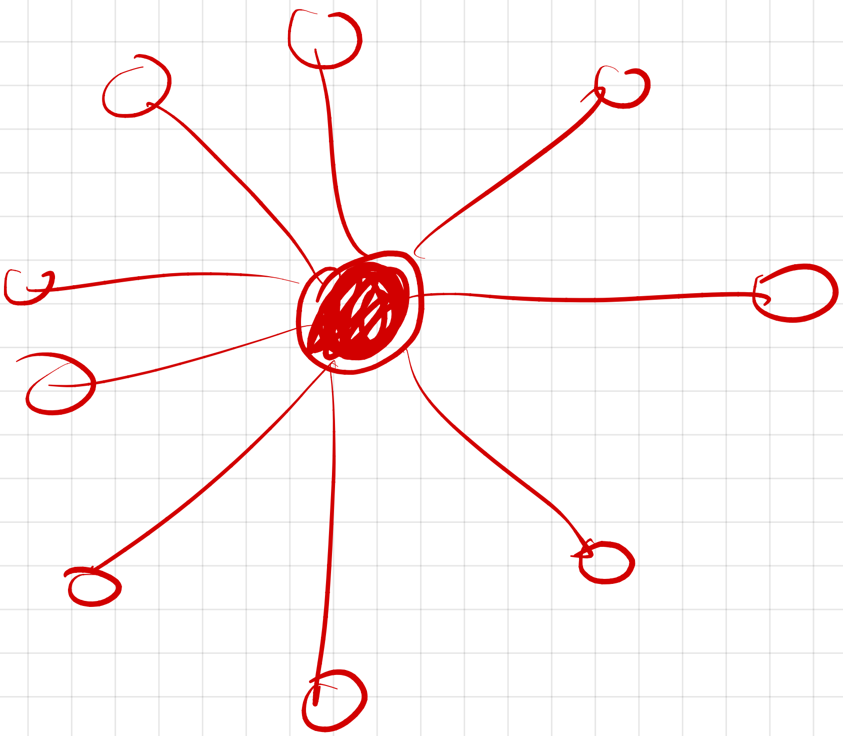
Euler Tour: Tour that visits every edge exactly once

all even degrees



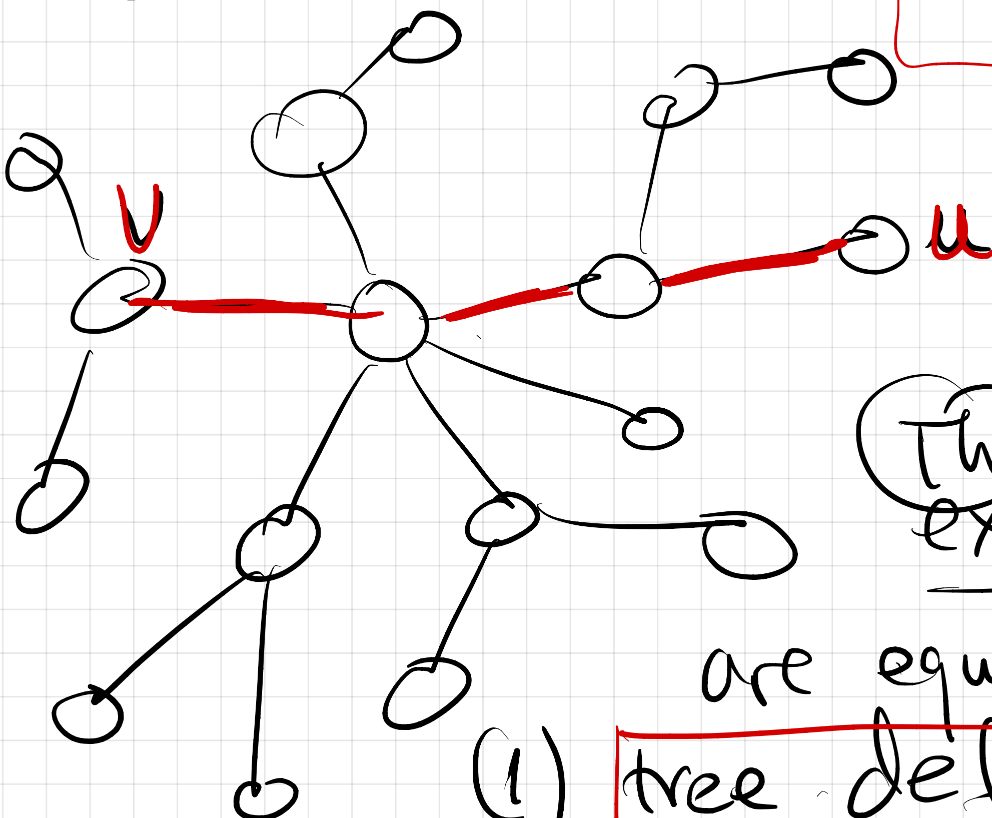
Vertex Cover: Find smallest set of vertices incident in all edges.

extreme: $|VC| = 1$



Tree

$T = G(V, E)$ = tree $\begin{cases} \text{connected} \\ \text{no cycles} \end{cases}$ &



exercise 1: $|E|$ in a tree is precisely $|V| - 1$

Th exercise 2 following statements are equivalent:

(1) $\boxed{\text{tree def}}$

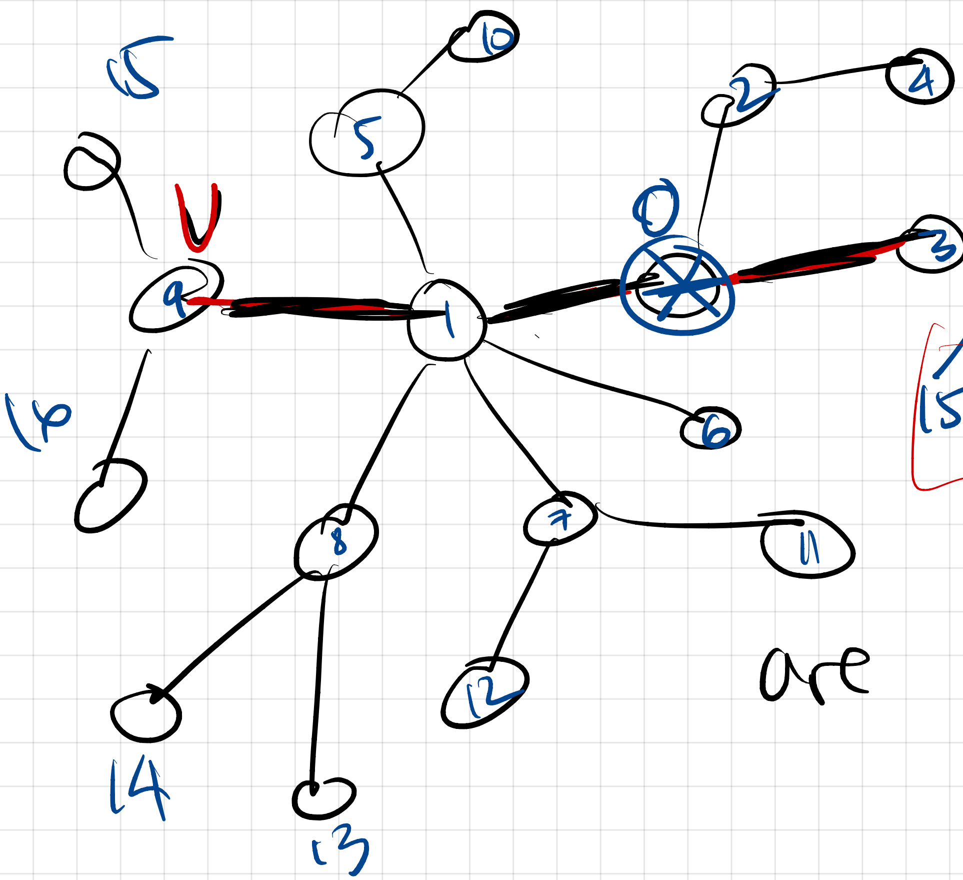
(2) any two vertices (u, v) connected with unique path

(3) T minimal connected (remove any edge $\Rightarrow$ disconnect)

(4) T max acyclic (add any missing edge $\Rightarrow$ cycle)

(5) connected & $|E| = |V| - 1$ || (6) acyclic & $|E| = |V| - 1$

pick root ⊛ → level 0



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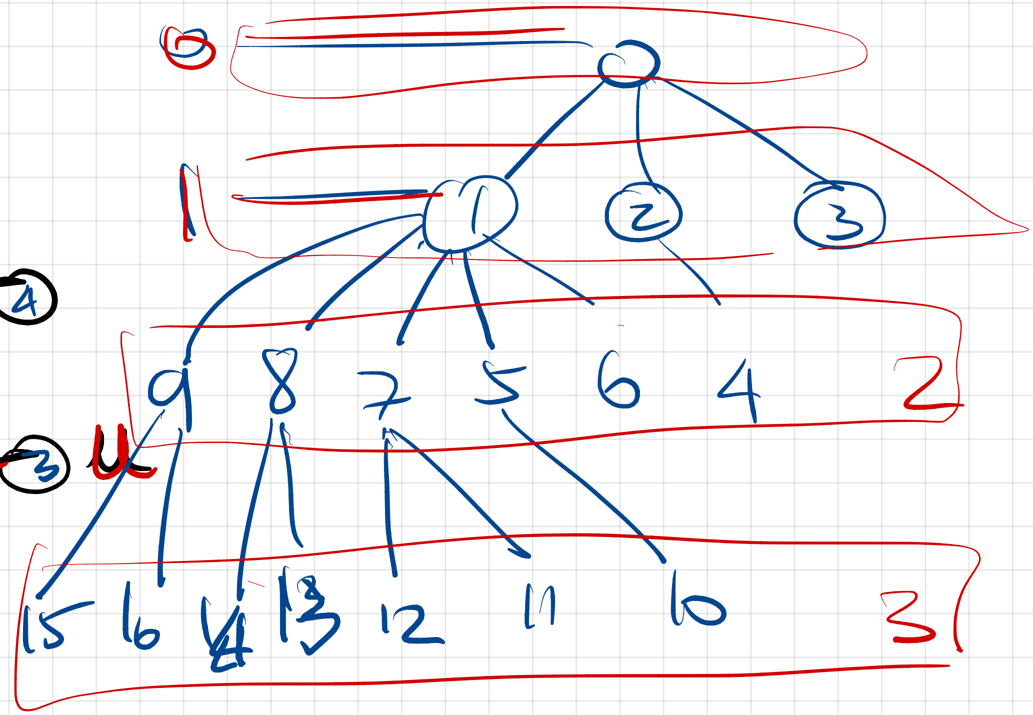

≃ BFS alg