CS1800
Discrete Structures
Fall 2019

Lecture 23
11/26/19

Last time

Graphs:

- representations
- traversals: BFS & DFS
- Handshake lemma

Today

- Finish handshake lemma
- Optional topics

Next time

- Exam review

Handshaking Lemma Proof 1: by induction over vertices.

B.C. $n=1$

- degree $=0$
- # vert w/ odd degree $=0$ even ✓

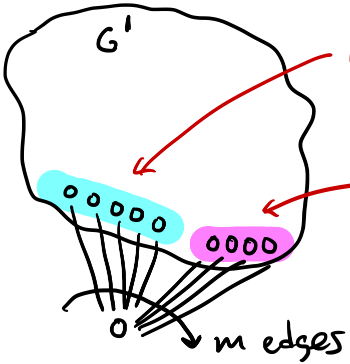
I.S.
- Assume true when $|V| = n = k$
- show true when $|V| = n = k+1$

- Consider any graph G with $k+1$ vertices

  let m be # incident edges

- Remove any one vertex and all its incident edges;
  what remains in a graph G' w/ k vertices → I.H. applies
  → # vertices w/ odd degree is even
- now consider returning the vertex and its m incident edges. Let i be the # of connected vertices w/ even degree in G' and $m-i$ the # with odd degree

- How does adding the vertex back change the number of vertices w/ odd degree?



i vertices in w/ <u>even</u> degree in G'

$m-i$ vertices w/ <u>odd</u> degree in G'

- How does the number of vertices w/ odd degree change once we add back the removed vertex?
- In G', # vertices w/ <u>odd</u> degree is <u>even</u>, by ind. hyp.

Two cases: ① m is <u>even</u>:

even to odd — odd to even — even — even

$$\text{change} = i - (m-i) = 2i - m$$

$\Rightarrow$ change is <u>even</u> ✓

② m is <u>odd</u>:

$$\text{Change} = 1 + i - (m-i) = 1 + 2i - m$$

new vertex — odd — even — odd

$\Rightarrow$ change is <u>even</u> ✓

Handshaking Lemma Proof 2: by induction over edges

B.C. Graph w/ 0 edges

$\Rightarrow$ all vertices have degree 0

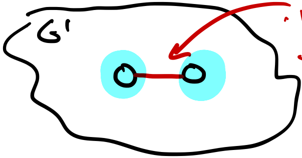
$\Rightarrow$ # vertices w/ odd degree is 0 which is even ✓

I.S. Assume true for $m=k$ edges;

Prove must true for $m=k+1$ edges.

- take any graph G w/ $k+1$ edges, remove any edge, obtaining graph G' w/ k edges. $\Rightarrow$ I.H. applies to G'
- Consider what happens when return edge back to graph.

$\Rightarrow$ only changes the degree of 2 vertices (the incident vertices).



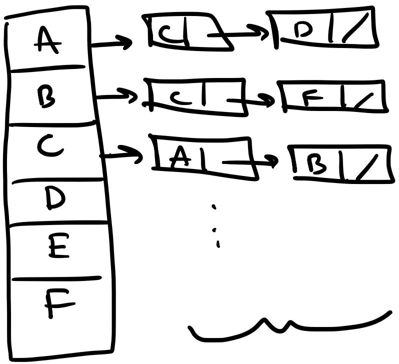
- reintroducing back
- affects degree of just blue vertices

cases for original degree of vertices in G':

		change in # verts. w/ odd degree
odd	odd	-2
odd	even	0
even	odd	0
even	even	$+2$

change always even ✓

Handshaking Lemma Proof 3: direct proof; adjacency list idea



$|V|$ $\quad$ $2 \cdot |E|$

$G = (V, E)$

$|V| = \#$ vert.

$|E| = \#$ edges.

(undirected)

total size =

$|V| + 2 \cdot |E|$

Consequence:

$$\sum_{v \in V} \deg(v) = 2 \cdot |E|$$

so, this must be even as well

even

even

$$\sum_{v \in V} \deg(v) = \sum_{\substack{v \in V \\ \text{where} \\ \deg(v) \text{ is even}}} \deg(v) + \sum_{\substack{v \in V \\ \text{where} \\ \deg(v) \text{ is odd}}} \deg(v) = 2 \cdot |E|$$

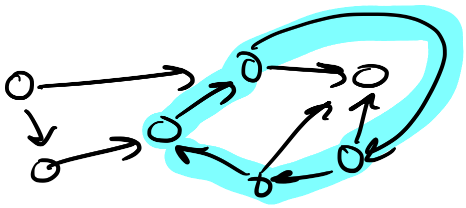
↳ · must be an even sum

· summing up all odd numbers

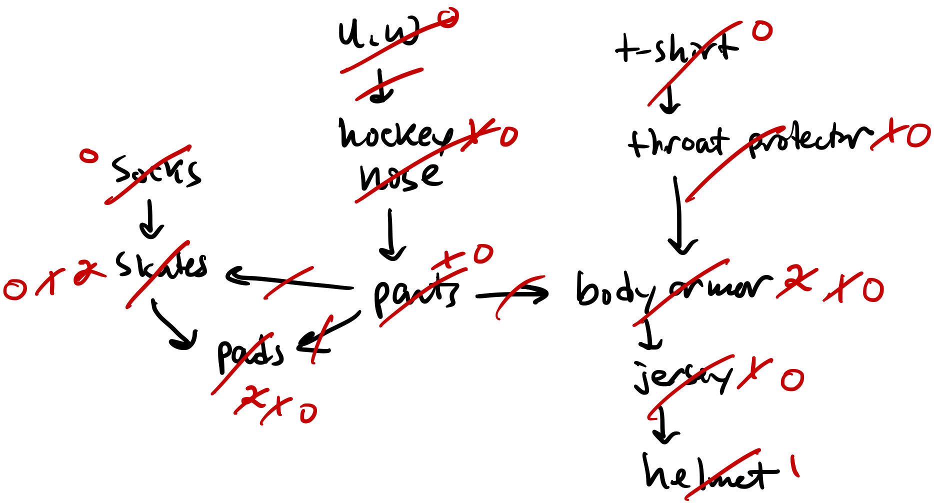
· to get an even sum, must sum up an even # of odd things.

⟹ # vert. w/ odd degree must be even

Topological Sort



Hockey Goalie getting dressed



u.w → hockey hose → pants socks → skates → pads t-shirt → t.p. → b.a → jersey → helmet