CS1800
Discrete Structures
Fall 2019

Lecture 22
11/22/19

Last time

- Finished growth of functions
- Start graphs
  - definitions
  - properties

Today

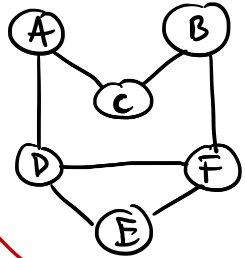
- more graphs
  - representations
  - traversals: BFS & DFS
  - Handshake lemma

Next time

- finish graphs

Graph Representation

① Adjacency Matrix



$$A = \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \\ F \end{array} \begin{array}{c} \begin{array}{cccccc} A & B & C & D & E & F \end{array} \\ \begin{pmatrix} 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 1 \\ & & \vdots & & & \\ & & & & & \end{pmatrix} \end{array}$$

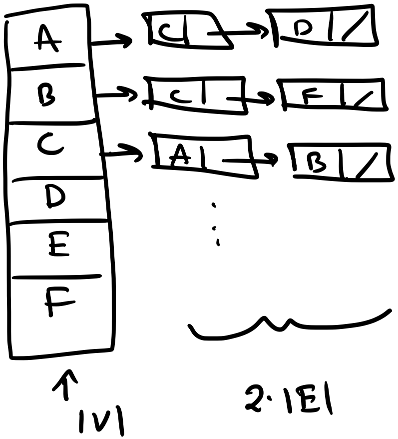
If a graph has n vertices, size of adjacency matrix is n^2

② Adjacency List Representation

$G = (V, E)$

$|V| = $ # vert.

$|E| = $ # edges.



(undirected)

total size $=$

$|V| + 2 \cdot |E|$

(directed)

$|V| + |E|$

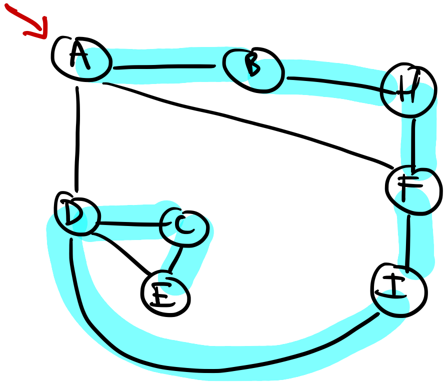
Fact: If a graph is <u>planar</u> (can be drawn in 2d w/o edges crossing), then

edges $\leq 3 \cdot$ #vertices $- 6$

Graph traversals

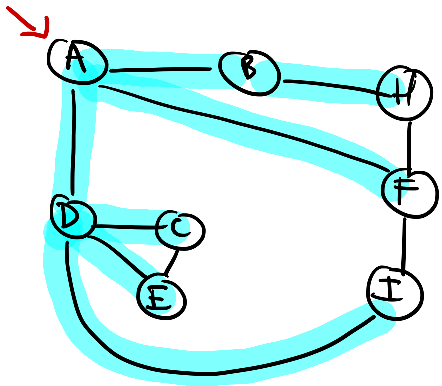
① Depth-first search

- "aggressive" search

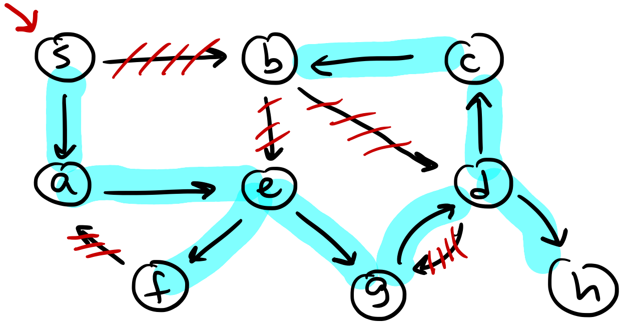


② Breadth-first search

- "timid" search



Depth-first search (carefully)

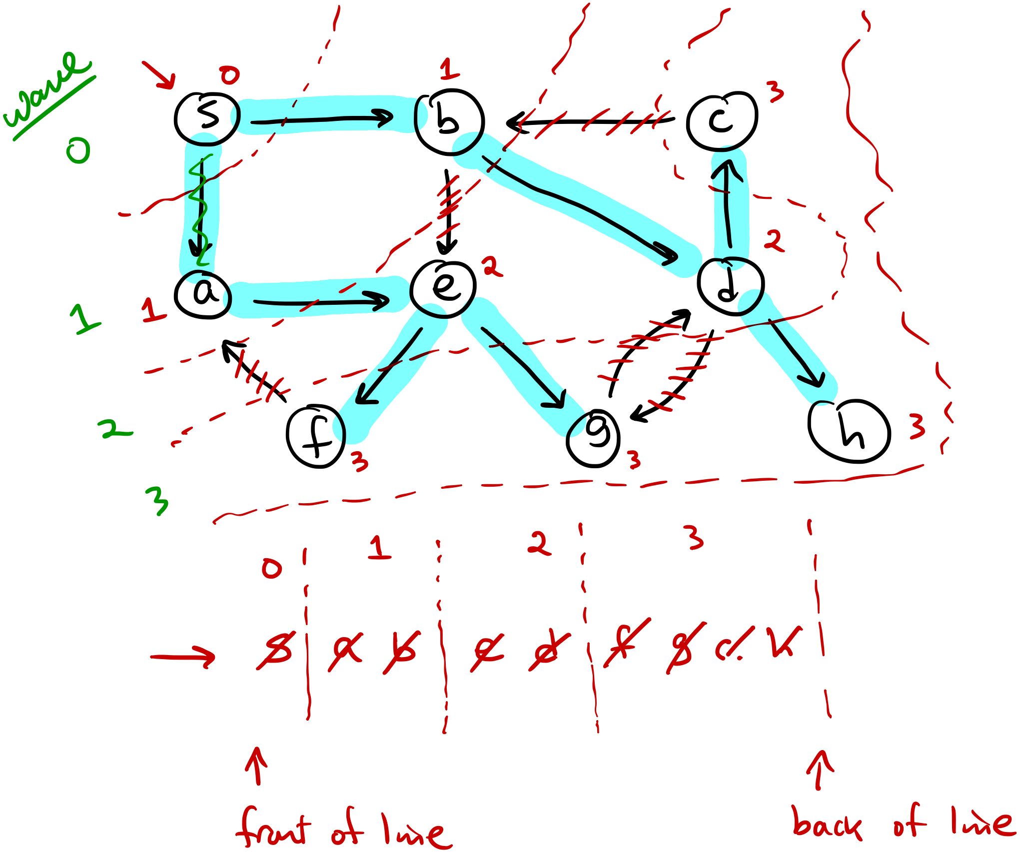


~~h~~
~~b~~
~~c~~
~~d~~
~~g~~
~~f~~
~~e~~
~~a~~
~~s~~

: the exploratory edges that form a depth-first tree

(edges that correspond to visiting a vertex for the first time)

Breadth-first search (carefully)

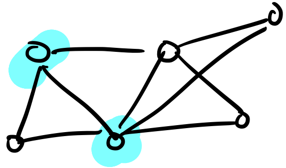


Graph Properties

Handshake Lemma:

- go to a party, shake some # hands
- # people who shake an <u>odd</u> # hands must be <u>even</u>

In terms of graphs...



vertices w/ <u>odd</u> degree must be <u>even</u>

- We will do <u>three</u> proofs, in <u>decreasing</u> order of complexity, to show that
  - you can solve problems in more than one way
  - thinking about the problem in the <u>right</u> way can make things easier.

- Proofs: (1) induction over vertices, (2) induction over edges, (3) direct proof, considering size of adjacency lists.

Pf

ind. over # vertices

B.C. $n=1$ o

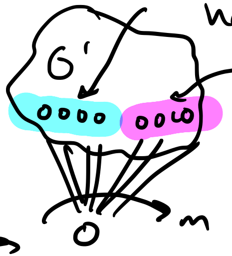
- degree $=0$
- # vert w/ odd degree $=0$ even ✓

I.S. assume true when $n=k$

show true when $n=k+1$

- Start w/ any graph w/ $k+1$ vertices
- remove 1 vertex and all of its incident edges
  - what's left is a graph w/ k vertices → I.H. applies
- Now consider returning vertex and its edges
- vertices in G' have even degree i

  vertices in G' have odd degree $m-i$



put back →

- How do the # vert w/ odd degree change?
- two cases:

m even: $i-(m-i) = \underbrace{2i}_{\text{even}} - \underbrace{m}_{\text{even}} =$ even change

m odd: $\underbrace{1}_{\text{odd}} + i-(m-i) = 1 + \underbrace{2i}_{\text{even}} - \underbrace{m}_{\text{odd}} =$ even change