REC 9: Probabilities

Due:

Instructions:

- HW instructions
- academic integrity and collaboration

Problem 1 [24 pts: (6 each)]:
A set of 6 cards contains two black cards numbered 1 and 2 and four red cards numbered 1, 2, 3, 4. Each of the questions below refers to uniformly selecting **one** card from the set of 6.

Please be mindful that you justify your answer algebraically (e.g. for conditional probabilities, be sure to reference $P(A|B) = \frac{P(A,B)}{P(B)}$)

i What is the probability that you draw a card whose value is 4?

ii What is the probability that you draw a card whose value is 4 given the card is black?

iii What is the probability that you draw a card whose value is 4 given the card is red?

iv Consider how the probability of choosing a four changes between parts i and ii. Tell if the probability increased or decreased and provide a sentence or two which provides an intuitive understanding of this change.

v Consider how the probability of choosing a four changes between parts i and iii. Tell if the probability increased or decreased and provide a sentence or two which provides an intuitive understanding of this change.

Problem 2 [24 pts: (6 each)]:
Gold prospecting is the search for new deposits of gold within the earth. If a parcel of land has a gold deposit, then there is a 90% chance that bits of gold will be found in one of its streams. However, gold can travel downstream into another parcel too! If a piece of land doesn't have a gold deposit, there is still a 40% chance that bits of gold are found in its streams. Only 30% of the parcels have gold deposits.
Let $D = 1$ be the event that a parcel has a gold deposit and $S = 1$ be the event that gold may be found in its streams.

i Describe, in english, each of the probabilities below.

(a) $P(D = 1)$

(b) $P(D = 1|S = 1)$

(c) $P(S = 0|D = 1)$

ii Compute $P(S = 1)$

iii What is the probability that a parcel of land has a gold deposit given that no gold may be found in its streams?

Problem 3 [24 pts: (6 each)]:
Suppose you tossed a coin 6 times. What is the probability that you obtained at least 1 tail?

Problem 4 [24 pts: (6 each)]:
Consider that two fair 3-sided die are thrown. As you would expect, the outcome of the two die are independent of each other.

We may express the outcome as a tuple: $(2, 3)$ which indicates that the first die is 2 while the second is 3.

- Let A be the event that the sum of the die is less than or equal to 3.
- Let B be the event that one die is odd while the other is even.

i What is the sample space, S, of the experiment?

ii What is the probability of each $x \in S$?

iii Express all of the elements in the sample space which satisfy A as a set.

iv Express all of the elements in the sample space which satisfy B as a set.

v Compute $P(A \cap B)$

vi Compute $P(A \cap B^c)$

Problem 5 [24 pts: (6 each)]:
Assume there are 365 days of the year and people are equally likely to be born on any day. What is the probability that everyone in a room has a unique birthday[1] if there are 10 people in a room? Do not simplify, just write out your expression. (HINT: tree method)

[1]For a fun follow-up, consider the following question:
If there are only 4 people in a room we'd expect the probability that two share a birthday to be near 0. However, if there are 300 people in a room we'd expect the probability that two share a birthday to be near 1. How many people are in the room when the probability that two people share a birthday is 50%? Use your answer to the problem above to build an estimate and then see what google has in store for you if you search 'birthday paradox'. Enjoy!