# REC 11: Probab, Expectation, Variance, Entropy

**Problem 1 bayes-doping**

The national flufferball association decides to implement a drug screening procedure to test its athletes for illegal performance enhancing drugs. 3% of the professional flufferball players actually use performance enhancing drugs. A test for the drugs has a false positive rate[1] of 2% and a false negative rate[2] of 4%[3].

A randomly selected player is tested and tests positive. What is the probability that she really does take performance enhancing drugs?

[1] the test detects drugs even though the individual didn't use drugs

[2] also known as a "missed detection rate": the test doesn't detect drugs even though the individual used drugs

[3] I agree that these terms are confusing ... but folks in different disciplines can't seem to agree on one set of terminology, see https://en.wikipedia.org/wiki/Confusion_matrix. If there's one term whose name is appropriate, its 'confusion' matrix.

**Problem 2 poisson-pop**

Back when Prof Higger taught high school physics, his students would sometimes entertain themselves with anything but physics. A favorite tactic was to make a popping noise[4] when I turned around to write anything on the board. Let us model the number of popping noises which occur in any single lesson as a Poisson distribution:

i The poisson distribution assumes that the events occur independently of each other. Identify some real-life high-school circumstance which would violate this assumption.

ii The poisson distribution assumes that the events occur at a constant rate. Identify some real-life high-school circumstance which would violate this assumption.

iii Suppose that in the past 10 days of classes, the following number of popping noises have been observed during each lesson:

$$7, 4, 8, 8, 2, 4, 2, 5, 4, 5$$

Estimate the rate parameter $\lambda$ of the Poisson distribution of pops per lesson.

iv Assuming the $\lambda$ estimate immediately above, compute the probability that a class has exactly 2 popping noises.

v One might also choose to model the popping noises as a Binomial Distribution (n= number of students in class and p=prob that they make a popping noise during class). Identify and explain which of the two Binomial assumptions you expect to be most problematic for this purpose.

[4] To make the noise, pull a hooked finger out of either side of your tightly closed lips. Its kind of brilliant really because no matter who makes the noise it sounds pretty similar so its hard to find out who did it. Honestly at the time it was super frustrating, though it makes me smile today. To be clear, however, this is **not** an invitation to make popping noises in class! :)

**Problem 3 exp-var-n-sided-die**

Compute the expected value and variance of each of experiments below. Assume that the die faces are numbered $1, 2, 3, \ldots, n$ for an n-sided die.

i a 3-sided die

ii Is the expected value of an 8-sided die greater than, less than or equal to a 3-sided die? Explain your response so it is easily understood by a non-technical reader in one sentence.

iii Is the variance of an 8-sided die greater than, less than or equal to a 3-sided die? Explain your response so it is easily understood by a non-technical reader in one sentence.

**Problem 4 exp-var-lotto**

Answer the questions below about the lottery tickets described. In these fictional lottery scenarios, there exist negative winnings. That is, somebody from the lottery comes to the ticket owner's door and demand that they are paid the losing values.

- Compute the expected value of each lotto ticket given
- Compute the variance and standard deviation of each lotto ticket given

i The random variable, $X$, represents the prize money of a 'Oh-No Lotto!' Ticket. It has the following distribution:

| X | P(X) |
|---|---|
| -$1000 | .001 |
| $50 | .01 |
| $10 | .1 |
| $0 | .889 |

Where $E[X]$ is the expected value of X, $\sigma_x^2$ is its variance and $\sigma_x$ is its standard deviation.

ii The random variable, $Y$, represents the prize money of a 'Steady-Going Lotto' Ticket. It has the following distribution:

| Y | P(Y) |
|---|---|
| $0 | .5 |
| $1 | .5 |

Compute the expected value of the winnings of this lottery ticket.

iii The random variable, $Z$, represents the prize money of a 'Shoot-For-The-moon Lotto' Ticket. It has the following distribution:

| Z | P(Z) |
|---|---|
| $1,000,000 | $10^{-6}$ |
| -$1 | $1 - 10^{-6}$ |

Compute the expected value of the winnings of this lottery ticket.

iv A friend of yours is interested in buying a ticket which, on average, makes them the most money. Also, given that your friend is recovering from a heart condition, you decide it would be wise to avoid surprises (both good and bad surprises!). Which ticket would you recommend? Give a mathematical reason why your selection is better than each of the other tickets.

**Problem 5 Problem 5 Breaking Eggs**

Please watch this important video before starting this question:

https://www.youtube.com/watch?v=naQeNNaZYoA

Jimmy and my boy Neil Patrick Harris started out with a carton of 12 eggs. 8 were boiled, 4 were raw. Note that the initial probability of getting a raw egg is 1/3, but that changes once they start cracking them on their faces.

Suppose a single person (could be Jimmy or NPH) cracks 3 eggs. What is the expected number of raw eggs that get smashed on that person's face?

**(A)** Calculate using E[] definition

**(B)** Calculate using E[] linearity over sums and indicator RV