HW8: Probabilities

Due:

Instructions:

- HW instructions
- academic integrity and collaboration

Problem 1 [24 pts: (6 each)]:
A class has $n = 250$ students, each either brings an umbrella to class or doesn't. Let us model the number of umbrellas brought to class as a Binomial Distribution with $n = 250$ and $p = .15 \approx 137/365 * .4$ (see footnote[1] for assumptions).

i Using the model, what is the probability that exactly two umbrellas are brought to class?

ii Using the model, compute the probability that 1 or more umbrellas are brought to class.

iii Identify the Binomial Model assumption which is most problematic in this umbrella context (No model is perfect, but this one has an egregious problem!). Clearly state this assumption, and why your experience with umbrellas / rain disagrees with it in one or two short, clear sentences.

iv (+2 pts extra credit): Given the assumption problem you identify immediately above, tell if part ii overestimates or underestimates the probability that no umbrellas are brought to class. Give a short, clear justification.

Problem 2 [24 pts: (6 each)]:

[1] Google says there are 137 rainy days a year in Boston, and we assume 40% of students bring an umbrella on rainy days

- Cars with flame paint jobs make up 1 percent of all cars in the neighborhood.
- Cars without flame paint jobs will drive above the speed limit 28 percent of the time.
- Despite how fast they look, drivers with paint flame jobs pamper their cars and only drive above the speed limit 4 percent of the time.

i Clearly define random variable(s) to represent whether a given car has a flame paint job and whether it is speeding.

ii Compute the probability that any car is not speeding (regardless of it is has a flame paint job or not). Justify your response by showing your algebraic expression before plugging in values.

iii Given a car was observed obeying the speed limit, what is the chance that it has a flame paint job? Justify your response by showing your algebraic expression before plugging in values. (Hint: feel free to re-use values computed from any previous part of the problem)

Problem 3 [24 pts: (6 each)]:
In a lottery, 6 balls are drawn from a pool of 49 balls painted with numbers 1 through 49.

i What is the probability that the numbers on the chosen balls are all squares of integers (e.g. 1, 4, 9, ...)?

ii What's the probability that all the six chosen numbers are composite (i.e. not prime)? (Since 1 is not considered prime, we will view 1 as a composite number for the purpose of this question).

Problem 4 Problem High/Low Betting

Two six-sided dice will be rolled independently. First, you have to guess if the sum is high or low. The sum is low if the sum is at most 6. The sum is high if the sum is at

least 7. Once you have guessed, the dice are rolled, each roll independent. If your guess is correct, you win. Otherwise, you lose.

i. Should you bet high or low?

ii. You think you can see a 4 on the face of the first die. Should you guess high or low? Explain.

iii. Suppose you guessed HIGH in part iii). The sum was 6 and you lost. You realize the die's face has faded over time, meaning it was equally likely you saw a 4 or a 5 (but no other). What is the probability the die was indeed a 4?

Problem 5 Problem 5 [Medium]**: Bayes**

Edgar tries to sell Ben on an analysis, LoopFinder, that announces if a program has an infinite loop. LoopFinder has a false positive rate of 5% (i.e. 5% chance LoopFinder announces there is an infinite loop given the program has none). Otherwise, LoopFinder is correct when a program does have an infinite loop.
To prove LoopFinder is worth his investment, Ben randomly generates a set of 98 programs and manually verifies 16 of them have infinite loops. Edgar then runs LoopFinder on those 98 programs.

i. In general, what is the probability LoopFinder announces there is an infinite loop in a program?

ii. On one program in the suite, LoopFinder announces there is an infinite loop. What is the probability that program actually has an infinite loop?