# **REC 7: Counting**

Due:

## Instructions:

- <u>HW instructions</u>
- academic integrity and collaboration

# Problem 1 [24 pts: (6 each)]:

A staircase has 10 steps. A person can take a normal step, to go up 1 step or a jump to go up two steps. How many different ways can a person climb the 10 steps? The order of their actions is significant (4 jumps and then 2 steps should be counted seperately from 2 steps and then 4 jumps).

#### Problem 2 [24 pts: (6 each)]:

A delivery driver has 10 **unique** packages to deliver.

- i How many ways can the delivery driver select 6 of these packages to deliver?
- ii How many ways can the delivery driver select 4 of these packages to discard while delivering the rest? Explain how this problem relates to the first subproblem.
- iii How many different routes may the delivery truck drive to deliver 8 of the 10 packages? A route is an ordered sequence of destinations. You may assume each package may only be delivered to its unique destination.
- iv Before leaving for the day, the delivery trucks are loaded with packages. How many ways can 120 **unique** packages be loaded into 10 delivery trucks where some trucks may have no packages?
- v How many ways can 120 **identical** packages be loaded into 10 delivery trucks where some trucks may have no packages?
- vi The delivery truck driver union is concerned that the workload is unequal. How many ways can 120 **identical** packages be loaded into 10 delivery trucks where each truck must have at least 5 packages?
- vii How many ways can 120 **unique** packages be loaded into 10 delivery trucks where each truck must have the same number of packages?

# Problem 3 [24 pts: (6 each)]:

A horse race has 12 horses.

- i How many different ways can the podium be arranged? (The podium has a spot for only the 1st, 2nd and 3rd place horse)
- ii How many different ways can the horses finish the race such that neither Grand Valor, Seabiscuit or Secretariat finish in first place?
- iii How many different ways can the horses finish the race such that one horse (Grand Valor) always beats both Seabiscuit and Secretariat?

## Problem 4 [24 pts: (6 each)]:

- i Lee wants to hang 8 of his 20 photos in an ordered line on the wall above his desk. How many ways can he do this?
- ii Lee wants to select 9 photos of his 20 to take on a trip. How many ways can he do this?
- iii Lee has decided to give his collection of 20 unique photographs to his three children. In how many ways can he partition his photo collection among this three children where it may be that some children get no photographs?
- iv Lee also has 500 dollars he wants to give his three children. In how many ways can he divide it among his three children? It may be the case that a child receives no money. Assume that Lee is distributing whole dollar bills to his children, each bill cannot be exchanged for coins.
- v Lee's children will be upset if he doesn't give each of them at least 50 dollars. How many ways can Lee partition his money so his children won't be upset?

# Problem 5 [24 pts: (6 each)]: (optional, no credit)

Make and solve your own counting problem! Good problems are fun, unambiguous and easily understood without much text. Consider using some subset of our counting moves (pro-tip: this is a great list to review for the exam)

- Sum Rule (really its principle of Inclusion / Exclusion)
- Product Rule (how many unique pairs can be made, one from set A and other from set B?)
- Permutations (ordering n distinct items)
- Combinations (selecting k distinct items from k)
- Partitions (how many ways can we split n identical items into k groups? AKA stars and bars)
- Counting "moves":
  - over-counting (if there are 40 toes in the room there are 4 people, see also derivation of combinations from permutations)
  - count-by-complement (number of items in whole set minus items we're not interested in)
  - count-by-partitioning (split the items we want to count into disjoint groups that are more easily counted)

If you've got a great problem you'd like to share, please email Prof Higger (mhigger@ccs.neu.edu) with one of the following permission statements written out, future students will benefit from your skills and humor :)

- You have permission to re-use and modify this problem for any purpose, please do cite me as the author of the original problem, my email is example@northeastern.edu<sup>1</sup>
- You have permission to re-use and modify this problem for any purpose, no citation needed.

<sup>&</sup>lt;sup>1</sup>Multiple emails welcome too, if you wrote in a team