HW6: Counting

Due:

Instructions:

- HW instructions
- academic integrity and collaboration

Problem 1 [24 pts: (6 each)]:
We define an "integer rectangle" as an rectangle with integer side lengths. For example a 3×5 rectangle is an integer rectangle while a 2.5×4 rectangle is not. We count a 3×5 rectangle separately from a 5×3 rectangle, they are not the same.

i How many integer rectangles have an area of 720? For example, 1×720, 720×1 and 72×10 should all be counted. (Hint: use prime factorization)

Problem 2 [24 pts: (6 each)]:

i Suppose 14 pigeons may land in 3 separate nests. How many pigeons, at least, are guaranteed to share the first nest?

ii Suppose 14 pigeons land in 3 separate nests. How many pigeons, at least, are guaranteed to be in the nest with the most pigeons?

iii What is the minimum number of students that must be assigned to a classroom with 14 tables to guarantee that there exists a table with at least 3 students? Justify your response with a sentence or two which is simple and easily understood by somebody who has never heard of the pigeonhole principle.

iv Some exam consists of 10[1] TRUE or FALSE questions, each is worth an equal amount of points. Assume that 123 students take this exam. Write **one sentence** which tells what the Pigeonhole Principle implies about the total exam scores of all students, no justification needed here. Your wording should be concise and unambiguous.

[1] Note that with 10 questions there are 11 unique scores a student could receive.

Problem 3 [24 pts: (6 each)]:
A group of 5 friends is sharing a collection of 15 different books. Each book must be given to exactly one person, and no book can be shared between two people.

i How many ways can we distribute the books among ourselves if each book is unique?

ii How many ways can we distribute the books among ourselves if each book is identical?

iii How many ways can we distribute the books among ourselves if each book is identical and one friend does not receive exactly 4 books (i.e., they receive more or less than 4 books)?

Problem 4 [24 pts: (6 each)]:
A survey contains 20 multiple-choice questions, each with four possible answers (A, B, C, or D). Each of the following questions refers to this survey.

i How many unique ways may a participant complete the entire survey?

ii How many unique ways may a participant 'complete' the survey if they leave at least one question unanswered?

iii How many ways can a participant respond if they select option A for exactly 8 of the questions and option B for the remaining 12 questions?

Problem 5 [24 pts: (6 each)]:
A multiple of a number is the product of that number with a natural number (excluding zero). So, the multiples of 5 are $\{5, 10, 15, 20, 25, \ldots\}$.

i How many integers from 1 to 1000 are multiples of 7?

ii How many integers from 1 to 1000 are multiples of 11?

iii How many integers from 1 to 1000 are multiples of 7 and 11?[2]

iv How many integers from 1 to 1000 are multiples of 7 or 11?

v How many integers from 1 to 1000 are multiples of 7 but not 11?

vi How many integers from 1 to 1000 are multiples of neither 7 nor 11?

Problem 6 [24 pts: (6 each)]:
A local high school soccer team has 20 players.

[2] Hint: the smallest positive value which is a multiple of both 7 and 11 is 77

i In how many ways can the coach choose 11 players to play?

ii A lineup describes the set of players which play together. A lineup consists of:

4 midfielders, 3 defenders, 3 attackers and 1 goalkeeper.

This team has:

7 midfielders, 6 defenders, 5 attackers and 2 goalkeepers

in total on their roster. In how many ways can the coach choose a lineup of players?

iii Now assume that one of the defenders, after a summer or practice, may play the attack position in addition to defense. In how many ways can the coach choose a lineup?