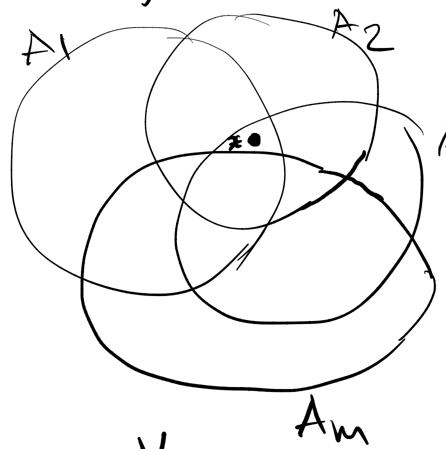
PIE general proof $|A_1 \cup A_2 \cup A_3 \cup \dots \cup A_m| =$



$= |A_1| + |A_2| \dots + |A_m|$ // |one set|

$- |A_1 \cap A_2| \dots - |A_i \cap A_j| \dots - |A_{m-1} \cap A_m|$ $-|\cap$ of $2|$

$+ |A_1 \cap A_2 \cap A_3| \dots + |A_i \cap A_j \cap A_k| + \dots + |A_{m-2} A_{m-1} A_m|$ $+|\cap$ of $3|$

$-$

$+$

$\vdots$

$+ (-1)^{m+1} |A_1 \cap A_2 \cap \dots \cap A_m|$ $\dots$ $|\cap$ of $m|$

Select $\forall x$ in $A_1 \cup A_2 \cup \dots A_m$. Its going to part of some sets

without lose of generality assume $x \in A_1 \cap A_2 \dots \cap A_n$ $(n \le m)$

$x \notin A_{n+1} \cup A_{n+2} \dots A_m$

<u>plan</u> $\boxed{\text{count } x}$ on RHS

$+ \binom{n}{1}$ $|A_1|$ $|A_2|$ $\dots$ $|A_n|$

$- \binom{n}{2}$ $|A_1 \cap A_2|, |A_1 \cap A_3| \dots |A_{n-1} \cap A_n|$

$+ \binom{n}{3}$ $|A_1 \cap A_2 \cap A_3| \dots |A_{n-2} \cap A_{n-1} \cap A_n|$

$\vdots$

$(-1)^{n+1}\binom{n}{n}|A_1 \cap A_2 \cap \dots A_n|$

$$count(x) = \binom{n}{1} - \binom{n}{2} + \binom{n}{3} \dots + (-1)^{n+1}\binom{n}{n}$$

Binomial Th: $\binom{n}{0} - \binom{n}{1} + \binom{n}{2} \dots + (-1)^n \binom{n}{n} = 0$

$$1 - count(x) = \overset{1}{\overset{\shortparallel}{\binom{n}{0}}} - \binom{n}{1} + \binom{n}{2} \dots - (-1)^n \binom{n}{n} = 0$$

$1 - count(x) = 0 \implies count(x) = 1$ ✓

PIE application: Derangement = permutation without fix points

$n=5$ (index sits on its own spot)

$$2\ 3\ 1\ 5\ 4 \quad \text{Derang.}$$

pos 1 2 3 4 5

$$3\ \boxed{2}\ 4\ 5\ 1 \quad \text{NOT DER } (\text{pos}(2)=2)$$

#Derangement(5) = ?

|all perm − all perm fixed points|

$A_1 = \{\text{permut } (1 \text{ fixed})\}$ $\boxed{1}\ \overbrace{\_\ \_\ \_\ \_}^{4!}$

$A_2 = \{\text{perm } (2 \text{ fixed})\}$ $\_\ 2\ \_\ \_\ \_$

$A_3 = \{\text{perm } (3 \text{ fixed})\}$ $\_\ \_\ 3\ \_\ \_$

$A_4 = \{\text{perm } (4 \text{ fixed})\}$ $\_\ \_\ \_\ 4\ \_$

$A_5 = \{\text{perm } (5 \text{ fixed})\}$ $\_\ \_\ \_\ \_\ 5$

$$= n! - \underbrace{|A_1 \cup A_2 \cup A_3 \cup A_4 \cup A_5|}$$

$$= |A_1| + |A_2| \ldots \quad 4!,\ 4!$$

$$- |A_1 \cap A_2| \ldots \quad 3!$$

$$+ |A_1 \cap A_2 \cap A_3| \ldots \quad 2!$$

$A_1 \cap A_2 = \{1\ 2\ \overbrace{\_\ \_\ \_}^{3!}\}$

$A_1 \cap A_2 \cap A_3 = \{1\ 2\ 3\ \overbrace{\_\ \_}^{2!}\}$