Binomial theorem (coef) $\Rightarrow$ Pascal $\triangle$ $\quad x, y \in \mathbb{R}$

$n=2$

$$(x+y)^2 = 1x^2 + 2xy + y^2 = \binom{2}{0}x^2 + \binom{2}{1}xy + \binom{2}{2}y^2$$

$$(x+y)^3 = 1x^3 + 3x^2y + 3xy^2 + 1y^3 = \binom{3}{0}x^3 + \binom{3}{1}x^2y + \binom{3}{2}xy^2 + \binom{3}{3}y^3$$

$$(x+y)^4 = 1x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + 1y^4 \qquad \binom{4}{2} = \frac{4!}{2!\cdot 2!} = \frac{3\cdot 4}{2}$$

$$\binom{4}{0}x^4 + \binom{4}{1}x^3y + \binom{4}{2}x^2y^2 + \binom{4}{3}xy^3 + \binom{4}{4}y^4$$

$$\binom{4}{1}x^{4-1}y^1 \qquad \binom{4}{2}x^{4-2}y^2 \qquad \binom{4}{3}x^{4-3}y^3$$

$(x+y)(x+y)(x+y)(x+y) \rightarrow$ 16 terms (incl repetion)

4 factors ()

$\rightarrow x^4$ — 1 way: x x x x

$\binom{4}{3}$ $\leftarrow$ $\boxed{x^3y^1}$ (3x, 1y) xxxy xxyx xyxx yxxx 4

3x choice

distinct terms $\boxed{x^2y^2}$ xxyy xyxy xyyx yxxy yyxx yxyx 6

$\binom{4}{2}$ choose 2 x

x^1y^3

y^4

how many ways to choose out of n

y choice in j paran ()

x choice in n-j paran ()

$$(x+y)^n = \sum_{j=0}^{n} \underbrace{\binom{n}{j}}_{\text{choose } j \text{ "y"}} x^{n-j} y^j = \sum_{j=0}^{n} \binom{n}{j} x^j y^{n-j}$$

n-j "x"

2^n terms (with repetitions)

$x=1 \quad y=1$

$$2^n = (1+1)^n = \sum_{j=0}^{n} \binom{n}{j} 1^{n-j} \cdot 1^j = \sum_{j=0}^{n} \binom{n}{j} = \binom{n}{0} + \binom{n}{1} + \dots + \binom{n}{n}$$

#subsets $|\,|=0$

#subsets $|\,|=1$

subsets all

$x=+1, y=-1$

$$0 = (1-1)^n = \sum_{j=0}^{n} \binom{n}{j} \boxed{1^{n-j}} \underline{(-1)^j} = \binom{n}{0} - \binom{n}{1} + \binom{n}{2} - \binom{n}{3} \dots (-1)^n \binom{n}{n}$$

$n=3 \qquad 1-3+3-1=0$

$n=4 \qquad 1-4+6-4+1=0$

$n=5 \qquad 1-5+10-10+5-1=0$

exercise: $\binom{n}{k} = \binom{n}{n-k}$ choose subset of k "in"

$\Leftrightarrow$ choose n-k stay out

n-k $\}$ k

$\binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1}$

case1 include last elem "n"

$\{ \underbrace{\overbrace{\_\ \_\ \_\ \_\ \_\ \_}^{k-1}}_{\text{from } \{1,2,\ldots n-1\}} \ n \}$ $\binom{n-1}{k-1}$

proof $\{1\ 2\ 3 \ldots \ n\}$ = SUM RULE

$\binom{n}{k}$ = # ways to choose a subset of k out $\{1,2,3, \ldots n\}$

case2 dont inclue "n"

$\{ \underbrace{\overbrace{\_\ \_\ \_\ \_\ \_\ \_}^{k \text{ elem}}}_{\text{from } \{1,2, \ldots n-1\}} \}$ $\binom{n-1}{k}$

verify (exercise) with factorials.

exercise

$$\binom{n}{k} = \binom{n}{n-k}$$

Pascal △

Binomial coeff

n		$(x+y)^n$
n=1	1 1	$(x+y)^1$
2	1 2 1	$(x+y)^2$
3	1 3 3 1	
4	1 4 6 4 1	
5	1 5 10 10 5 1	
6	$\boxed{1 - 6 + 15 - 20 + 15 - 6 + 1}$	

$$\binom{5}{3} = \binom{4}{2} + \binom{4}{3}$$

$$\binom{5}{1} = \binom{4}{0} + \binom{4}{1}$$

exercise

$$\binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1}$$

$*$

$x=p \quad y=1-p$ $\quad$ p = prob of success (coin flip)

1-p = prob failure

$$1=(p+(1-p))^n = \sum_{j=0}^{n} \underbrace{\binom{n}{j} p^{n-j}(1-p)^j}_{\text{prob(exactly } j \text{ successes)}}$$ Binomial distribution

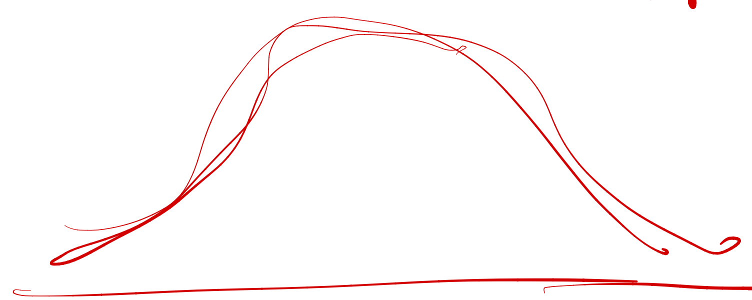
$p=\frac{1}{3}$ chance of sum

$n=100$ coin flips

$p=\frac{1}{2}$:

$$\frac{\sum_{j=0}^{n} \binom{n}{j}}{2^n}$$

dist on pizza ; **Binomial** $n=100, p=\frac{1}{2}$



gauss approxim (mean $\frac{n}{2}$, var $\frac{n}{4}$)

good approx for $n \geq 9$