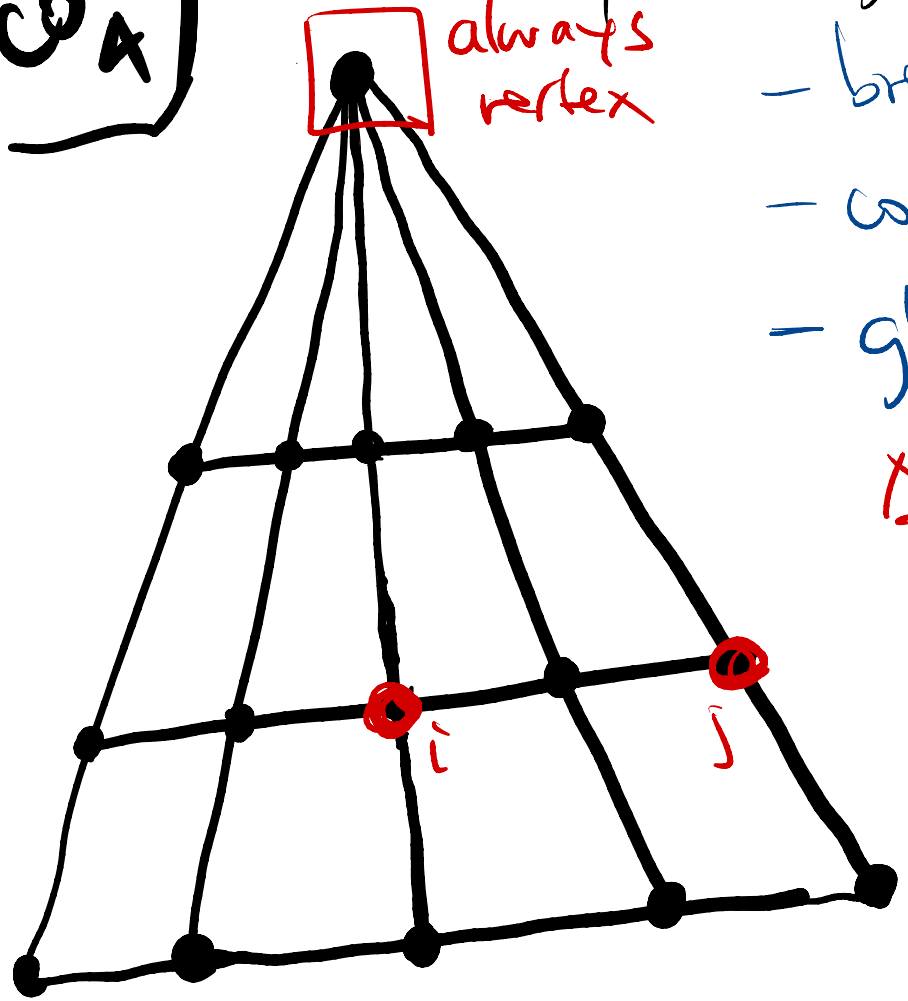
COUNT 4

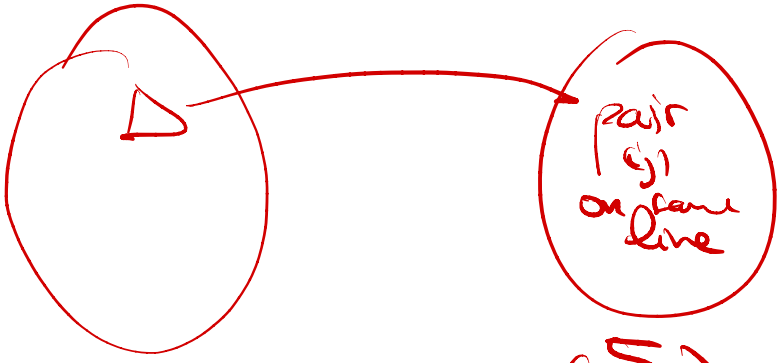
How many triangles?



Divide & Conquer:

- break pb into choices / subpbs
- count/solve those subpbs/choices
- glue them together

Δ = chose 2 dots on same horizontal line

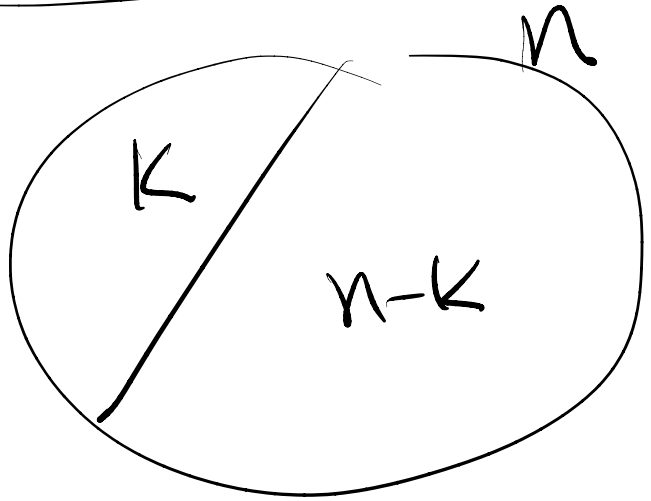


pairs (i,j) on same line = 3 (choose a line) × $\binom{5}{2} = 3 \cdot \frac{5!}{2!\,3!}$ (choose 2 dots out of 5 (no order, no repet))

(Th) $\sum_{k=0}^{n} \binom{n}{k} = 2^n$

$$\binom{n}{k} = \frac{n!}{(n-k)!\,k!}$$

$$\sum_{k=0}^{n} \frac{n!}{k!(n-k)!} = 2^n$$



choosing k-set

≡ choosing remaining n-k set

$$\binom{n}{k} = \binom{n}{n-k}$$

$$\frac{n}{k!\,(n-k)!} = \frac{n!}{(n-k)!\,(n-(n-k))!}$$

PB4 Counting with cases (Sum Rule disjoint OR PIE not disjoint)

T W V G preferable

36 = 13 witches + 4 warlocks + 12 vampires + 7 goblins
distinguishable

3 discussion sessions A, B, C ⇒ partition of all, 12 each

(A) How many ways to make sessions?

$$\binom{36}{12}\binom{24}{12}\binom{12}{12}$$

choose session A — choose session B — session C

(B) Sessions with restriction: all warlocks same session.

warlocks in B

$$\binom{32}{8}\binom{24}{12}\binom{12}{12} + \binom{32}{12}\binom{20}{8}\binom{12}{12}$$

non W — non W

A session W

+ warlocks in "C"

© session ©= 2G + 2V + 4T + 4W

want © to break into 2 groups ("red" "blue") of size 4 each

Constraints:

goblins ∉ Blue ; vampires ∉ Red.

---

3 cases: #G in Red group. → 0, 1, 2

red 0G : $\binom{8}{4}_{\text{red}} \times \binom{4^{\text{nonGV}}+2^{\text{V}}}{4}_{\text{blue}}$ (8 → nonGV)

red 1G : $\binom{2}{1}_{\text{goblin}} \times \binom{8^{\text{nonGV}}}{3}_{\text{red}} \times \binom{5^{\text{nonGV}}+2\text{V}}{4}_{\text{blue}}$

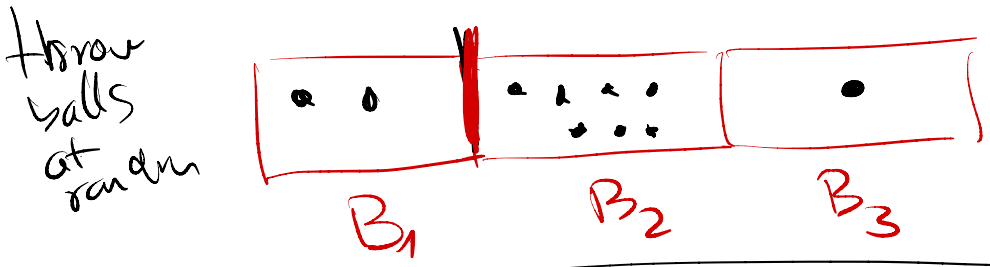
2G : $\binom{2}{2} \times \binom{8\text{nonGV}}{2} \times \binom{6\text{nonGV}+2\text{V}}{4}_{\text{blue}}$

BALLS INTO BINS (STARS AND BARS)

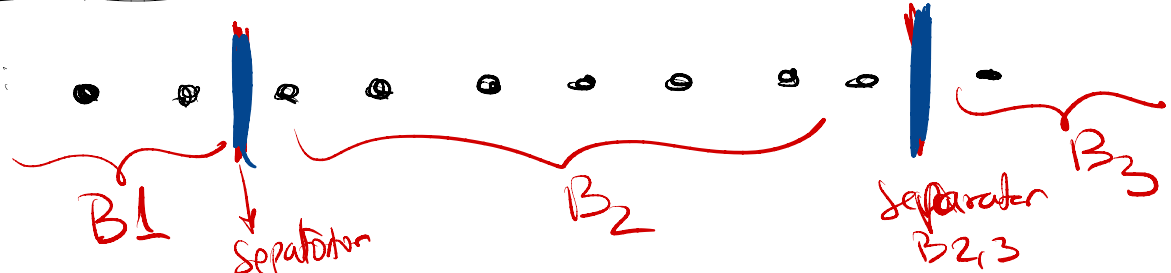
#ways to place n identical balls in k bins

(not distinguishable) (distinguishable)

ex: $n=10$ candies (identical) distribute to $k=3$ children $c_1, c_2, \ldots c_k$



Solution: $k-1$ separators



$n+k-1$ items (balls n, separators $k-1$)

2, 7, 1 ● ● | ○ ○ ○ ○ ○ ○ ○ | ○ ○ ⇒ #ways

$\binom{n+k-1}{n} = \binom{n+k-1}{k-1}$ choose $k-1$ spots "|"

4, 0, 6

1, 9, 0

0, 0, 10

Rules for proper counting

- ITEMS are distinguishable / NOT
- REPETITIONS / NOT (REP)
- ORDER / NOT ORDER