- with repetition: $S = \{A, B, C, D\}$

sequence of 5 letters (rep. allowed): A B A A C

A A C D C

$\underset{4 \text{ possi}}{\underline{A, B, C, D}}$ $\underset{4 \text{ possb}}{\underline{A, B, C, D}}$ ___ ___ $\underset{4 \text{ possib}}{\underline{A, B, C, D}}$

- any combination (sequence) valid

$$\# \text{sequences} = 4 \times 4 \times 4 \times 4 \times 4 = 4^5$$

- License plates 8 spots

$\underset{26}{\_}\ \underset{36}{\_}\ \underset{36}{\_}\ \_\ \_\ \_\ \_\ \underset{36}{\_}$ 8

$S = \{\text{Letters (cap)} + \text{all digits}\} \Rightarrow 36$

- $\boxed{\text{no repetitions}}$ $S = \{a, b, c, d, e\}$

$\Rightarrow$ max # spots $= |S|$

5 spots

(longest sequence)

$\boxed{\text{permutation (of all elements in } S)}$ $= 5!$

$\underset{5}{\_\_}$ $\underset{4}{\_\_}$ $\underset{3}{\_\_}$ $\underset{2}{\_\_}$ $\underset{1}{\_\_}$ 120

(one is used) (two are used) the only one left

no repetitions

a	~~b~~ e	c	d	e
b	c	d	a	e
e	d	b	c	a
⋮				

} 5-plets sequence of length 5

no repetitions $S = \{a, b, c, d, e\}$ $n = 5$

sequences of length $k = 3$ $k \le n$

$5 \cdot 4 \cdot 3$

$\underset{n}{\underline{\quad 5 \quad}}$ $\underset{n-1}{\underline{\quad 4 \quad}}$ $\underset{n-2}{\underline{\quad 3 \quad}}$

$\boxed{\begin{matrix}(a\ b\ c) \\ (b\ a\ c)\end{matrix}}$ diff

$(c\ a\ b)$

$(d\ a\ c)$

$(e\ d\ b)$

$\frac{5!}{2!}$ $\frac{5!}{(5-3)!}$

general $S = \{1, 2, \ldots n\}$

want sequence of length $k \le n$

$\underline{\quad}\times\underline{\quad}\times\underline{\quad}\times\underline{\quad}\times\underline{\quad}$

spot 1: n

spot 2: $n-1$ (one used)

$n-2$

spot k: $n-k+1$

$n \times (n-1) \times \ldots \times (n-k+1)$

choose seq of k IN ORDER out of n $= P(n,k) = \frac{n!}{(n-k)!} = \frac{n \cdot (n-1) \cdots (n-k+1)(n-k) \cdots 1}{(n-k)(n-k-1) \cdots 1}$

- no repetition. no-order

n items: 1, 2, ... n
K spots

$$\underset{\text{spot1}}{\underline{n}} \times \underset{\text{spot2}}{\underline{(n-1)}} \times \underset{\text{spot3}}{\underline{(n-2)}} \times \underline{\quad} \quad \underline{\quad} \times \underset{\text{spotK}}{\underline{(n-k+1)}}$$

remaining $n-K$

write it down (output) AS A SET

$n=5 \quad K=3$

sequen

a	b	c	d, e
b	a	c	d, e
c	a	b	
a	c	b	
b	c	a	
c	b	a	

$6 = \frac{5!}{2!}$

$\frac{n!}{(n-k)!}$

SETS

one set {a, b, c}

{b, a, c}

{c, a, b}

{a, c, b}

{b, c, a}

{c, b, a}

K! occurences of set {a, b, c}

different sets $\times$ K! = # diff seq

$P(n,k) = \frac{n!}{(n-k)!}$

diff sets $= \frac{n!}{(n-k)!\,k!}$

choose k items (no order, no repetition) out of n

"n choose k" $= \boxed{\binom{n}{k}} = C(n,k) = nCk$

choose a subset of size k out of a set of n elements.

set $= \{1, 2, 3, \dots n\}$

$\binom{n}{k}$ = # different subsets of size k (out of n)

$$2^n = \underbrace{\binom{n}{0}}_{\#\text{subsets size}=0} + \underbrace{\binom{n}{1}}_{\#\text{subs. size } 1} + \underbrace{\binom{n}{2}}_{\#\text{subsets size } 2} + \underbrace{\binom{n}{3}}_{\#\text{subsets size } 3} + \dots + \underbrace{\binom{n}{n-1}}_{\#\text{subsets size } n-1} + \underbrace{\binom{n}{n}}_{\#\text{subsets size}=n}$$

$\emptyset$

$\{1\}$, $\{2\}$, $\{3\}$, …, $\{n\}$

$\{1,2\}$, $\{1,3\}$, …, $\{n-1,n\}$

$\{1,2,3\}$, $\{1,2,4\}$, …, $\{n-2,n-1,n\}$

$\{1,2,\dots n-1\}$, …, $\{2,3,\dots n\}$

1: $\{1:n\}$

COUNT 1

Two decks of cards are mixed together, total 104 cards where each card appears exactly twice. How many distinct permutations are there of all 104 cards?

global: $\frac{104!}{\#\text{double counted}} = \frac{104! \text{ all permute}}{\underbrace{2 \cdot 2 \cdot 2 \cdots 2}_{52}}$

2^{52}

$2! = 2$

Constructive

__ __ __ $\overset{3}{X}$ __ $\overset{5}{X}$ __ __ __ 104 spots

choose 2 spots for (A♦ two) $\binom{104}{2} = \frac{104 \cdot 103}{2}$

choose 2 spots for (♣ two) $\rightarrow \binom{102}{2} = \frac{102 \cdot 101}{2}$

choose 2 spots for (3♥ two) $\rightarrow \binom{100}{2} = \frac{100 \cdot 99}{2}$

⋮

choose (last rem 2 spots) for las card $\times 2$ $\rightarrow \binom{2}{2} = \frac{2 \cdot 1}{2}$

COUNT 2

A) How many ways to place a rock, knight and bishop on a chess board (8x8) such that no two of them are on the same row or column?

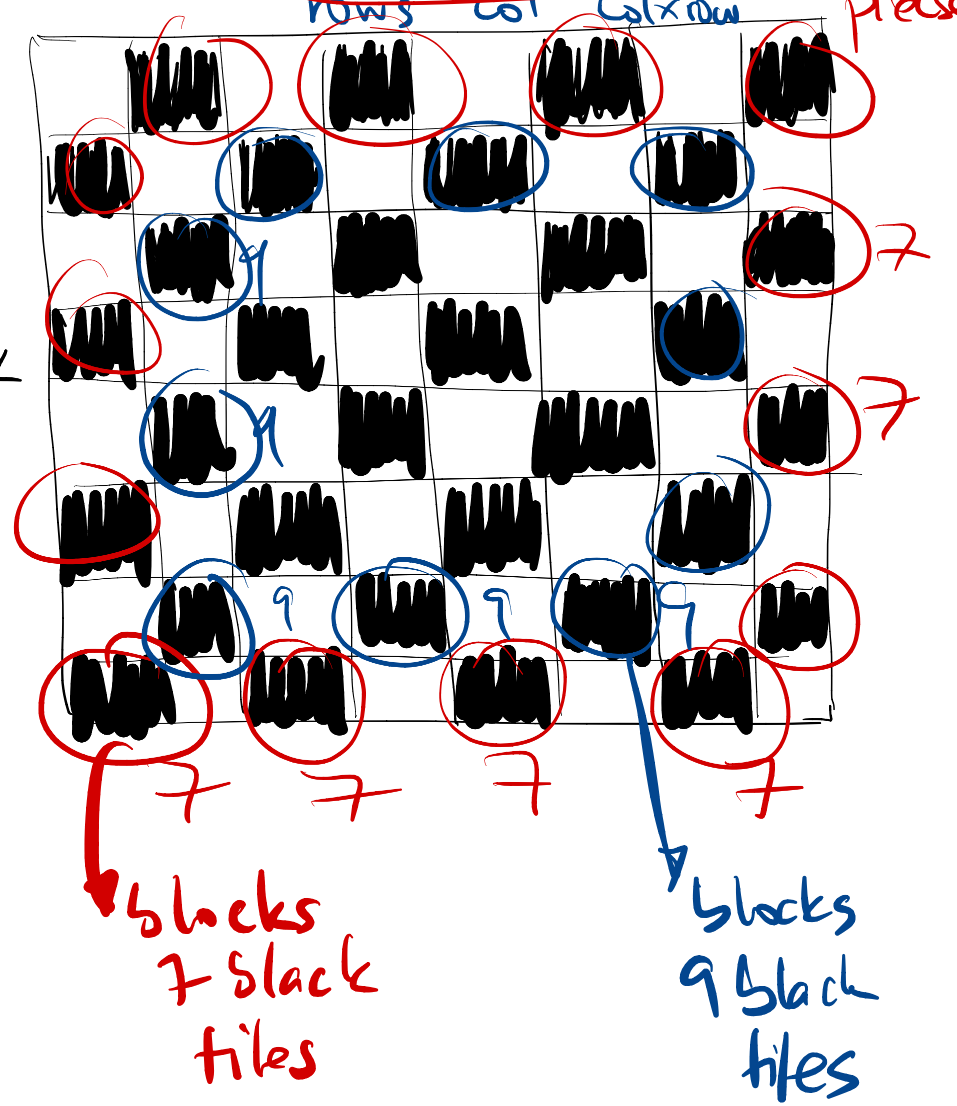
$\boxed{\binom{8}{3}\binom{8}{3}\cdot 3! \times} \; 3!$ please

rows, col, colxrow, pos

$8^2 \times 7^2 \times 6^2$?

B*) How many ways to place 2 bishops that do not attack each other? [bishop attack on diagonals]

exercise



C) ~~***~~ 3 Bishops

COUNT 3

How many different permutations of "MISSISSIPPI"?

naive: $M_1, S_1, S_2, S_3, S_4, P_1, P_2, I_1, I_2, I_3, I_4$

11 objects: Permute them $\Rightarrow$ 11!

same $\begin{cases} M_1 S_1 I_1 S_2 I_2 I_3 \ldots \\ M_1 S_2 I_2 S_1 I_3 I_4 \ldots \end{cases}$ WRONG!

Sol 1: $\dfrac{\text{count all permutations } 11!}{\text{\# permutations make same word}}$

$\boxed{\text{\# permutations } S_i I_i M S_i \ldots}$ $4 \times 4 \times 2$?

all those give same word. $4! 4! \cdot 2!$?

SIMSISIPISP =

2 5 7 9

I-pos = {2, 5, 7, 9} 4! to place 4-Is on those spots.

Constructive

- place the S : S S S S 1 choice

  seq 4

- place I-s in between

  S S S S

  Box 1 Box 5

  4 I-s into 5 boxes

  $\Downarrow$

  seq of of 8

  S+I

- place P-s in between

  2 P-s in 9 boxes

  P+S+I

- place M $\Rightarrow$ 11 choices

- product of choices $\rightarrow$? exercise next week