

REC7: Induction, Series, PHP

Problem 1 Counting Fashion

- i. A math professor wears for each class a combination of : one of their three hats (red, blue, green); one of their pants (black, blue, white, green); one of their shirts (white, red, black); and one of their pair of shoes (black, red, blue, white). They never wear combinations with three items of the same color. How many different combinations are wearable?
- ii. The professor is teaching 70 lectures for the term. Can they wear a different combination for each lecture during one term?
- iii. If they do wear a different combination each lecture, show that they must use either the red or the blue shoes that term.
- iv. There are 4 terms for the year. Show that at least one combination will be used at least 3 times during the year.
- v. How many valid combinations have blue hat or shoes ?

Problem 2

Suppose a set of 8 numbers are selected from the set:

$$\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14\}$$

Show (and justify) that two of the selected numbers must sum to 15. (Hint: Pigeonhole)

Problem 3

- i How can the product rule be used to find the number of functions from a set with m elements to a set with n elements?
- ii How many functions are there from a set with five elements to a set with 10 elements?

Problem 4

Prove that convex polygon with n sides has $n(n - 3)/2$ diagonals.

Problem 5 Who owns the fish?

There are 5 houses in 5 different colors in a row. In each house lives a person with a different nationality. Each of the 5 owners drinks a certain type of beverage, smokes a certain brand of cigar, and keeps a certain pet. No two owners have the same pet, smoke the same brand of cigar, or drink the same beverage. One of them owns a fish.

Other facts:

- The Brit lives in the red house.
- The Swede keeps dogs as pets.
- The Dane drinks tea.
- The green house is on the immediate left of the white house.
- The green house's owner drinks coffee.
- The owner who smokes Pall Mall rears birds.
- The owner of the yellow house smokes Dunhill.
- The owner living in the center house drinks milk.
- The Norwegian lives in the leftmost house.
- The owner who smokes Blends lives next to the one who keeps cats.
- The owner who keeps the horse lives next to the one who smokes Dunhill.
- The owner who smokes Bluemasters drinks beer.
- The German smokes Prince.
- The Norwegian lives next to the blue house.
- The owner who smokes Blends lives next to the one who drinks water.

The question is.... Who owns the fish???

Problem 6 ★ optional, no credit
Prove that $n^{1/n} < 2 - 1/n$ for integers $n > 1$