# HW4: Induction

**Due:**

**Instructions:**

- HW instructions
- academic integrity and collaboration

## Problem 1 Problem 1 Sequences, Series, Summations

**i.** Give a general formula for the sequence $a_0 = -2, a_1 = 6, a_2 = -18, a_3 = 54, a_4 = -162, a_5 = 486, \ldots$
Then calculate the series of the first $n$ terms

**ii.** Compute a close form as function of $n$ $S(n) = \sum_{k=4}^{n}(3k-1)$, using series calculations
Then prove it by induction over $n$.

**iii.** ★ (optional, no credit) Compute a close form as function of $n$. Dont use induction; instead subtract $2S(n) - S(n)$ term by term to get a familiar series

$$S(n) = \sum_{k=1}^{n} k \cdot 2^k$$

**iv.** (optional, no credit) Prove the formula obtained from previous exercise by induction over $n$

## Problem 2 Problem 3 Induction simple proofs

**i.** Let $a$ and $n$ be positive integers, with $a$ odd. Prove by induction over $n$ that $2^{n+2} | (a^{2^n} - 1)$

**ii.** Prove by induction over $n$ that any set of size $n$ has $2^n$ subsets

**Problem 3 [24 pts: (6 each)]:**

The diagonal of a square matrix refers to all elements in a line from the top-left to the bottom-right of the matrix. For example, in the $3 \times 3$ matrix below, all diagonal entries are 1 where off-diagonal entries are 0:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Note that the only entry in a $1 \times 1$ matrix is on the diagonal.

The upper-diagonal of a matrix are all entries which are on the diagonal or above. For example, in the $3 \times 3$ matrix below, all upper-diagonal entries are 1 where non-upper-diagonal entries are 0:

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

Use induction to show that an $n \times n$ matrix has $n(n+1)/2$ upper-diagonal entries.