

REC 3: Logic, Sets

Due:

Instructions:

- [HW instructions](#)
- [academic integrity and collaboration](#)

Problem 1 [24 pts: (6 each)]:

- Using the statements below, write the conditional $P \rightarrow Q$ as well as its contrapositive, converse & inverse in english.
 - P = it is raining
 - Q = my dog is wearing a rain jacket
- Which of the four statements generated above are logically equivalent to others? (and what does it mean to be logically equivalent?)
- Assume that, when it rains, I always put a rain jacket on my dog. Is the statement $P \leftrightarrow Q$ true? Explain why or why not.

Problem 2 [24 pts: (6 each)]:

When my son was young, he and the dog had a bad habit of sharing chew toys!¹



Let us define the following predicates on chew toys x

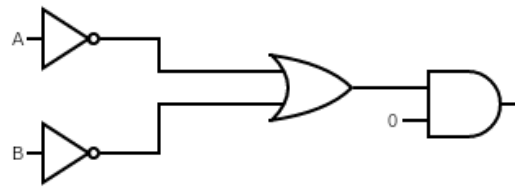
- $D(x)$ = my dog is chewing on toy x right now
- $S(x)$ = my son is chewing on toy x right now

Translate each of the following logic statements into plain english, easily understood by a non-technical reader.

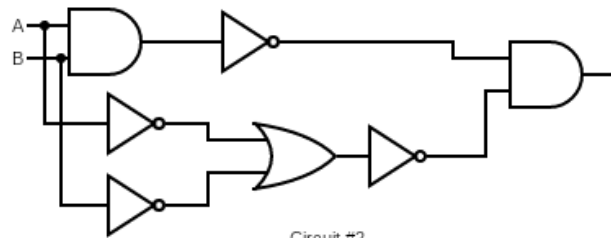
- i $\exists x D(x)$
- ii $\forall x S(x)$ (he's an impressive kid :))
- iii $\forall x S(x) \rightarrow D(x)$ (definitely gross, maybe cute too?)
- iv $\forall x \exists y S(y) \rightarrow D(x)$
- v $\exists y \forall x S(y) \rightarrow D(x)$

¹Maybe it builds immunity ... he's made it a few years after the fact so it couldn't have been that bad, right?

Problem 3 [24 pts: (6 each)]:



Circuit #1



Circuit #2

- i Write out logical expressions representing each of the two circuits. Show that they are equivalent using the laws of logical equivalence.
- ii There are many other circuits that would be equivalent to these two. Draw one that uses three AND gates, one NOT gate, and no other gates. Write its logical expression.

Problem 4 [24 pts: (6 each)]:

i Express the set:

$$S = \{n \in \mathbb{N} | (-11 \leq n) \wedge (n \bmod 7 = 4) \wedge (n < 10)\}$$

by explicitly writing each item in a set (e.g. $\{1, 2, 3\}$). Assume that the set of natural numbers is $\mathbb{N} = \{0, 1, 2, 3, \dots\}$.

ii Express the set of all integers whose absolute value is less than 10 using set builder notation.

Problem 5 [24 pts: (6 each)]:
 Shade the indicated regions of the following Venn diagrams.

