HW3: Sets

Due:

Instructions:

- HW instructions
- academic integrity and collaboration

Problem 1 [24 pts: (6 each)]:
Consider the bit string representation of sets A and B:
$A = \{$`paul, george`$\}$
$B = \{$`ringo, george`$\}$

U	`john`	`paul`	`ringo`	`george`
A	0	1	0	1
B	0	0	1	1
$A \cup B$				
$A \cap B$				
A^C				

For each of sets below:

- complete the empty row to the table which gives the bit string representation of the set
- tell which logical operator (AND, NOT, XOR, OR) of the bit string representations of A, B yield the same bit string representation of the set

i $A \cup B$

ii $A \cap B$

iii A^C

Problem 2 [24 pts: (6 each)]:

i Express the set:

$$S = \{n \in \mathbb{Z} | n \in \mathbb{N} \text{ and } (-11 \leq n) \text{ and } (n < 10)\}$$

by explicitly listing each item in a set (e.g. $\{1, 2, 3\}$). We assume that $0 \in \mathbb{N}$ above.

ii Express the set B of all integers whose square is either 25 or 36 using set builder notation.

iii Express the set B immediately above by listing.

Problem 3 [24 pts: (6 each)]:

Simplify each of the following expressions by writing a sequence of equalities. Each equality should be labelled with the justifying algebraic law (see logic_set_identities.pdf). Please do not use multiple laws in a single step or use the set difference operator at all. U indicates the universal set which contains all elements.

i $((A^C \cap B) \cup (A^C \cap B^C))^C$

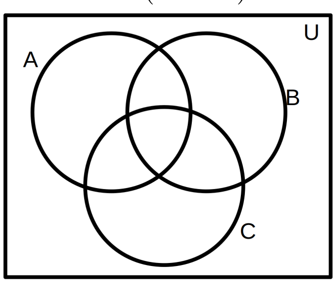
ii $(A^C \cap B^C)^C \cap U$

iii $(A \cup A) \cap (B \cup A^C)$

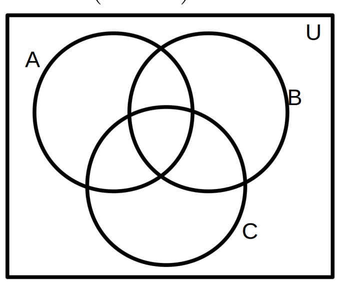
Problem 4 [24 pts: (6 each)]:

Shade the indicated regions of the following Venn diagrams.

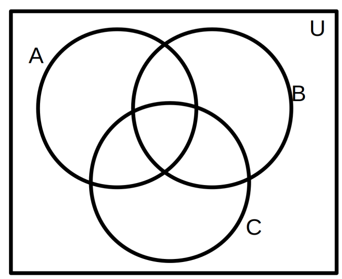
$A \cup (B - C)$



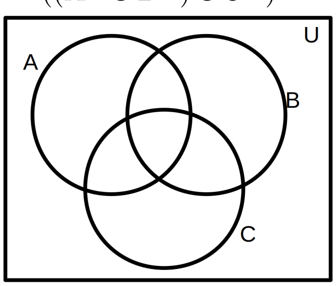
$(A \cup B) - C$



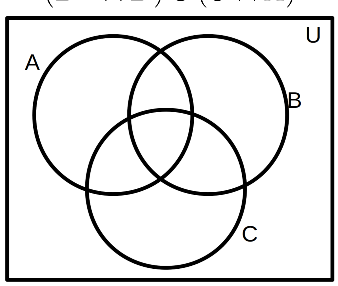
$A^C \cap B^C$



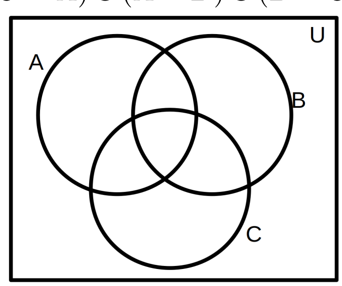
$((A^C \cup B^C) \cup C^C)^C$



$(B^C \cap B) \cup (C \cap A)$



$(C - A) \cup (A - B) \cup (B - C)$



Problem 5 [24 pts: (6 each)]:
Consider the subsets $A = \{1, 2, 3, 6, 9\}$ and $B = \{3, 5\}$ of the universal set U $= \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$. Express each of the following sets as an explicit list (e.g. $\{3, 5, 6\}$).

i $\{2x + 1 \in U \mid\ x \in A\}$

ii $\{x \in A \mid\ x \text{ is odd}\}$

iii $\{x \in B \mid\ 2x \in U\}$

iv $A \cap B$

v $A \cup B$

vi $A - B$

vii $\overline{A \cap \overline{B}}$

viii $A \triangle B$

Problem 6 [24 pts: (6 each)]:
A multiple of a number is the product of that number with a natural number (excluding zero). So, the multiples of 5 are $\{5, 10, 15, 20, 25, \ldots\}$.

i How many integers from 1 to 1000 are multiples of 7?

ii How many integers from 1 to 1000 are multiples of 11?

iii How many integers from 1 to 1000 are multiples of 7 and 11?[1]

iv How many integers from 1 to 1000 are multiples of 7 or 11?

v How many integers from 1 to 1000 are multiples of 7 but not 11?

vi How many integers from 1 to 1000 are multiples of neither 7 nor 11?

[1] Hint: the smallest positive value which is a multiple of both 7 and 11 is 77