Problem 1 [24 pts: (6 each)]:

For each sub-part below, please use a 8-bit two's complement representation:

- i Give the decimal (base-10) for 2's complement value $(1000000)_2$
- ii Give the decimal (base-10) for 2's complement value $(11110011)_2$
- iii Give the decimal (base-10) for 2's complement value $(01111111)_2$
- iv When adding the following values (from immediately above) $(1000000)_2 + (11110011)_2$ in 8-bit two's complement, does an overflow occur? Why or why not?
- v When adding the following values (from immediately above) $(0111111)_2 + (11110011)_2$ in 8-bit two's complement, does an overflow occur? Why or why not?

Problem 2 [24 pts: (6 each)]:



Use the following predicates:

- circle(x) is True when x is a circle
- rect(x) is True when x is a rectangle
- star(x) is True when x is a star
- shade(x) is True when x is shaded in
- $next_to(x, y)$ is True when the squares containing x and y touch (i.e. they're immediate neighbors left-right, top-bottom, or they share a corner diagonally).

to determine if each of the statements below is true or false. For statements which are true, justify your answer by describing all relevant elements (maybe all of them). For statements which are false, justify your answer by giving a single counter-example.

i star(d)

ii $\exists x \operatorname{star}(x) \land \operatorname{shade}(x)$

- iii $\forall x \operatorname{star}(x) \land \operatorname{shade}(x)$
- iv $\forall x \operatorname{circle}(x) \rightarrow \neg \operatorname{shade}(x)$

Problem 3 [24 pts: (6 each)]:

Complete the truth table below for the statement

 $\neg[(x \land y) \lor z]$

Use the intermediate columns to build the statement incrementally.

x	y	z	$x \wedge y$	$(x \land y) \lor z$	$\neg[(x \land y) \lor z]$
F	F	F			
F	F	Т			
F	Т	F			
F	Т	Т			
Т	F	F			
Т	F	Т			
Т	Т	F			
Т	Т	Т			

Problem 4 [24 pts: (6 each)]:

Consider the documents needed to apply to prove one's identity in a credit card application. Express each sentence using logical operations \neg , \land , \lor and the propositional variables b, d, and p defined below. If there's any confusion about the precise meaning of the sentence, you're welcome to use the given table, which provides the same information in another format. The use of the word "or" in the sentences below always means inclusive or.

b	Applicant has birth certificate
d	Applicant has driver's license
р	Applicant has passport

i The applicant must present a birth certificate, a driver's license, or a passport.

b	d	р	Desired Outcome
F	F	F	\mathbf{F}
\mathbf{F}	\mathbf{F}	Т	Т
\mathbf{F}	Т	\mathbf{F}	Т
\mathbf{F}	Т	Т	Т
Т	F	F	Т
Т	\mathbf{F}	Т	Т
Т	Т	\mathbf{F}	Т
Т	Т	Т	Т

ii Applicant must present a birth certificate or both a driver's license and a passport.

b	d	р	Desired Outcome
F	F	F	F
\mathbf{F}	\mathbf{F}	Т	F
\mathbf{F}	Т	\mathbf{F}	F
\mathbf{F}	Т	Т	Т
Т	F	F	Т
Т	\mathbf{F}	Т	Т
Т	Т	\mathbf{F}	Т
Т	Т	Т	Т

iii The applicant must present at least two of the following forms of identification: birth certificate, driver's license, passport.

\mathbf{b}	d	\mathbf{p}	Desired Outcome
F	F	F	F
\mathbf{F}	\mathbf{F}	Т	\mathbf{F}
\mathbf{F}	Т	\mathbf{F}	\mathbf{F}
\mathbf{F}	Т	Т	Т
Т	F	F	F
Т	\mathbf{F}	Т	Т
Т	Т	\mathbf{F}	Т
Т	Т	Т	Т