CS1800
Discrete Structures
Fall 2019

Lecture 4
9/17/19

Last time

- Recap & finish two's comp.
- Logic gates

Today

- Logic & Boolean Algebra

Next time

- Finish Logic
- Combinatorics

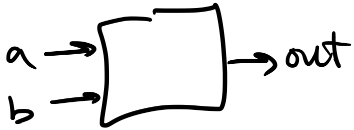
Announcements

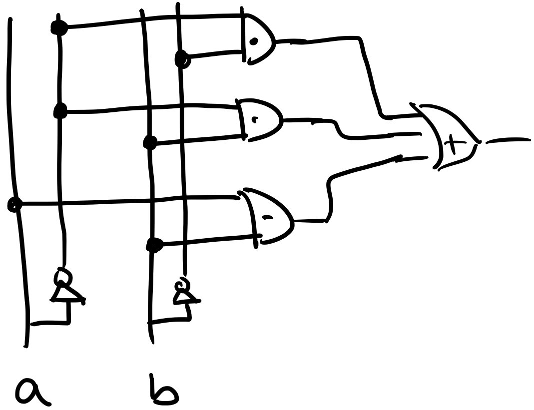
- HW1 Due tonight – Gradescope
  - late HW not accepted, because...
- HW1 solutions posted Wed
- HW2 now available on website
- If you have DRC accommodations, <u>please</u> let us know
- Recitation quiz 1 is <u>next week</u>
  - circuits & conversions

DNF Construction Example

$(x-2)(x+3) = x^2+x-6$

a	b	out
0	0	1
0	1	1
1	0	0
1	1	1





$$\equiv (\neg a \wedge \neg b) \vee (\neg a \wedge b) \vee (a \wedge b)$$

$$\stackrel{?}{\equiv} \neg a \vee b$$

a	b	$\neg a \vee b$
0	0	1
0	1	1
1	0	0
1	1	1

✓

Laws of (Boolean) Logic/ Algebra

	"normal algebra"	Boolean algebra
comm. laws	$a+b=b+a$	$a \vee b \equiv b \vee a$
	$a \cdot b = b \cdot a$	$a \wedge b \equiv b \wedge a$
dist. laws	$a \cdot (b+c) = a \cdot b + a \cdot c$	$p \wedge (q \vee r) \equiv p \wedge q \vee p \wedge r$
		$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
ident.	$a \cdot 1 = a$	$p \wedge T \equiv p$
	$a + 0 = a$	$p \vee F \equiv p$
De Morgan		$\neg (p \wedge q) \equiv \neg p \vee \neg q$
		$\neg (p \vee q) \equiv \neg p \wedge \neg q$

Example (Logical Equivalence)

$$(\neg a \wedge \neg b) \vee [(\neg a \wedge b) \vee (a \wedge b)]$$

$$b \to p$$
$$\neg a \to q$$
$$a \to r$$

$$\equiv (\neg a \wedge \neg b) \vee [b \wedge (\neg a \vee a)]$$

$$\equiv (\neg a \wedge \neg b) \vee [b \wedge T]$$

$$\equiv (\neg a \wedge \neg b) \vee b$$

$$\equiv (\neg a \vee b) \wedge (\neg b \vee b)$$

$$\equiv (\neg a \vee b) \wedge T$$

$$\equiv \neg a \vee b \quad \checkmark$$

$$a \vee b \equiv b \vee a$$

$$a \wedge b \equiv b \wedge a$$

$$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$$

$$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$$

$$p \wedge T \equiv p$$

$$p \vee F \equiv p$$

$$\neg (p \wedge q) \equiv \neg p \vee \neg q$$

$$\neg (p \vee q) \equiv \neg p \wedge \neg q$$

a	b	out	out'
0	0	1	0
0	1	1	0
1	0	0	1
1	1	1	0

DNF construction: focus on 1's of output

$$out \equiv (\neg a \wedge \neg b) \vee (\neg a \wedge b) \vee (a \wedge b)$$

$\vdots$ ← lots of Boolean algebra

$$\equiv \neg a \vee b$$

Apply DNF construction to out', set

$$out' = a \wedge \neg b$$

$$out \equiv \neg out' \equiv \neg(a \wedge \neg b)$$

$$\equiv \neg a \vee \neg\neg b$$

$$\equiv \neg a \vee b$$

By construction, $out \equiv \neg out'$

Example 2

a	b	out	out'
0	0	0	1
0	1	1	0
1	0	1	0
1	1	0	1

DNF: $out \equiv (\neg a \wedge b) \vee (a \wedge \neg b)$

$out' \equiv (\neg a \wedge \neg b) \vee (a \wedge b)$

but...

$$
\begin{aligned}
out &\equiv \neg\, out' \\
&\equiv \neg \left[(\neg a \wedge \neg b) \vee (a \wedge b) \right] \\
&\equiv \neg (\neg a \wedge \neg b) \wedge \neg (a \wedge b) \\
&\equiv (\neg\neg a \vee \neg\neg b) \wedge (\neg a \vee \neg b) \\
&\equiv (a \vee b) \wedge (\neg a \vee \neg b)
\end{aligned}
$$

CNF Form

conjunctive normal form

Formal Logic : if-then → implications
if-and-only-if

① Implies : $A \Rightarrow B$ "A implies B"

A : "hot outside"

B : "not many people outside"

implication: $A \Rightarrow B$ "if hot outside, then not many people outside"

converse : $B \Rightarrow A$ "if not many people outside, then hot outside"

* converse not necessarily true

Contrapositive: $\neg B \Rightarrow \neg A$ "if many people outside, then not hot outside"

* implication and its contrapositive are logically equivalent

Implications chain together

$$A \Rightarrow B, \quad B \Rightarrow C \quad \Longrightarrow \quad A \Rightarrow C$$

Fri afternoon

traffic is bad

traffic is bad

at least 45 mins to get home

Fri aft.

45 min to get home

First-order Logic

if and only if

① Consider : "any object is a square iff it is a rectangle with equal length sides"

$P \iff Q$ $\qquad P =$ "any object is a square"

what does that mean?

* need variables and predicates

$$\underbrace{Square}_{\text{predicate}}(\underbrace{x}_{\text{variable}}) \iff Rectangle(x) \wedge EqualSides(x)$$

② Quantifiers

$\exists$ = "there exists"

$\forall$ = "for all"

$\exists x : 2x = 4$

$\forall x : \text{Even}(2x)$

additive inverses always exist

TRUE $\forall x \exists y : x + y = 0$

FALSE $\exists y \forall x : x + y = 0$

TRUE $\exists y \forall x : x \cdot y = 0$

$y = 0$