Office hours 9/14

digits = {0, 1, 2, 3}

Binary representation

Base 16

A B C D E F

10 11 12 13 14 15

Base 2	Base 4	Base 10	Base 16
11011	123	27 +	16+11 = 1B +
10001	101	17	16+1 = 11
101100 = ?	230	44	? = 2C

power base expansion

$1\cdot 2^5 + 1\cdot 2^3 + 1\cdot 2^2 + 0\cdot 2^1 + 0\cdot 2^0$

$= 32 + 8 + 4 = 44$ ✓

power base expansion

$2\cdot 16^1 + C\cdot 16^0 =$

$32 + 12 = 44$

$123_{(4)} = 1\cdot 4^2 + 2\cdot 4 + 3 =$

$16 + 8 + 3 = 27$ ✓

$230_4 = 2\cdot 4^2 + 3\cdot 4 + 0 = 32 + 12 = 44$ ✓

Binary 6bits $\boxed{\_\ \_\ \_\ \_\ \_\ \_}$

How many # we can represent?

How many possibilities in binary 6bits? 64

000000
000001
000010
...
111111

→ Which 64 numbers do we represent? [0 : 63]

{0, 1, 2, ... 63}

maybe [100 : 163]

signed range [−32 : 31] {−32, −31, −30, ..., −1, 0, 1, 2, ..., 31}

rep = two complement

3 shirts S_A, S_B, S_C

2 pants P_1, P_2

4 hats H_A, H_B, H_C, H_D

Q: how many ways to dress up?

Product Rule

all choices can be combined
(every choice valid with any other choice
- no restriction)

$3 \times 2 \times 4 = 24$ possib.

$(S_A, P_1, H_A), (S_A, P_1, H_B) \ldots$

$(S_A, P_2, H_A) \quad (S_A, P_2, H_B) \ldots$

6 bits

2 pos 0/1	2 pos 0/1	0/1	0/1	0/1	0/1

Product Rule? all choices are valid Yes

$\#$ possib $= 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^6$.

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4 digits base 3

$d \in \{0, 1, 2\}$

3 poss 0/1/2	0/1/2	0/1/2	0/1/2

Product Rule? are all choices valid with any other choices? YES

$3 \times 3 \times 3 \times 3 = 3^4 = 81$

how many possib base 3 4 digits

- ending with "2"

$\boxed{}\ \boxed{}\ \boxed{}\ \boxed{2}$

$27 = 3 \times 3 \times 3 \times 1$

- ending with "2" or "0"

$\boxed{}\ \boxed{}\ \boxed{}\ \boxed{0/2}$

$3 \times 3 \times 3 \times 2$

base 10 5 digits

$d \in \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$\square \quad \square \quad \square \quad \square \quad \square$

$\underset{0 \to 9}{10} \times \underset{0 \to 9}{10} \times 10 \times 10 \times \underset{0 \to 9}{10}$

10^5 possibilities.

unsigned Range $\{0, 1, 2, \ldots, 99999\}$

$[0 : 99999]$

Size of the range

$|Range| = 10^5 - 1$ why?

$\underset{Range}{99999} + 1 = 1\underbrace{00000}_{10^5}$

what is max in Range (unsigned)

base 10 k digits.

$\underset{k-1}{\underline{d_{k-1}}}$ __ __ __ _ _ __ d_3 d_2 $\underset{1}{\underline{d_1}}$ $\underset{0}{\underline{d_0}}$

Generic "d_{k-1} d_{k-2} .... d_1 d_0"

power expansion: $d_{k-1} \cdot 10^{k-1} + d_{k-2} 10^{k-2} + \ldots + d_1 \cdot 10^1 + d_0$

max possible: $9 \cdot 10^{k-1} + 9 \cdot 10^{k-2} \ldots 9 \cdot 10^1 + 9$

$= \boxed{(10-1)} (10^{k-1} + 10^{k-2} + 10^{k-3} - + 10^1 + 10^0)$

$$10^k - \cancel{10^{k-1}} +$$
$$\cancel{10^{k-1}} - \cancel{10^{k-2}}$$
$$\cancel{10^{k-2}} - \cancel{10^{k-3}}$$
$$\cancel{- \; - \; -}$$
$$\cancel{10^1} - 10^0$$

telescope

$$10^k - 10^0 = 10^k - 1$$

signed binary $\boxed{6 \text{ bits}}$ _ _ _ _ _ _

$2^6 = 64$ possibilites

Range = [-32 : 31]

"∈" in a set belongs to

mechanism/procedure to map each value ∈ [-32:31] to string of 6 bits $\boxed{\text{TWO'S COMPLEMENT}}$

- unsigned mechanism = stays. $\boxed{0:31}$ ✓

$\boxed{0}$ 0 1 1 0 1 = 13 ✓

positive

8 4 1

use the other 5 bits for unsigned value

-1 = ? negative ⇒ first bit = 1

decimal value

1 1 1 1 1 1 ?

"complement of 1" = ?

1 1 1 1 1 0 ? Not sure

later

1 1 1 1 1

-1 = ? 1 1 1 1 1 ?1

add

+1 = 0 0 0 0 0 1

0 = 0 0 0 0 0 0

(1)

-2 = ? 1 1 1 1 1 ?0

+2 = 0 0 0 0 1 0

0 = 0 0 0 0 0 0

-32 (min in range) = ? ?1 0 0 0 0 0

+32 unsigned = 1 0 0 0 0 0

unsigned (not sign bit) 0 0 0 0 0 0

Exercise: two's complement works like this

$\underline{1}\ \underline{0}\ \underline{1}\ \underline{1}\ \underline{1}\ \underline{0}$

power expansion $-1\cdot 2^5+0\cdot 2^4+1\cdot 2^3+1\cdot 2^2+1\cdot 2^1+0\cdot 2^0$

has the first power with negative sign.

$$\boxed{-9 = ?\quad 1\ 1\ 0\ 1\ 1\ 1}$$

$$+9 = \quad 0\ 0\ 1\ 0\ 0\ 1$$

$$0\ 0\ 0\ 0\ 0\ 0$$

2's comp power exp

$110111 =$

$-1\cdot 2^5 + 1\cdot 2^4 + 0\cdot 2^3 + 1\cdot 2^2 + 1\cdot 2 + 1$

$-32 + \underbrace{16 + \quad + 4 + 2 + 1}$

$= -32 + 23 = -9$