

Office hours 9/14 → digits = $\{0, 1, 2, 3\}$
 binary representation

base 16
 A B C D E F
 10 11 12 13 14 15
 base 16

Base 2	Base 4	Base 10	Base 16
11011	123		
10001	101		
<u>101100</u> = ?	<u>230</u>	<u>44</u>	
power base expansion			
$1 \cdot 2^5 + 1 \cdot 2^3 + 1 \cdot 2^2 + 0 \cdot 2^1 + 0 \cdot 2^0$			
$= 32 + 8 + 4 = 44 \checkmark$			

$$\begin{array}{r}
 16+11 = 1B \\
 16+1 = 11 \\
 \hline
 ? = 2C
 \end{array}$$

power base expansion

$$\begin{array}{r}
 2 \cdot 16^1 + 0 \cdot 16^0 = \\
 32 + 12 = 44
 \end{array}$$

$$123_{(4)} = 1 \cdot 4^2 + 2 \cdot 4 + 3 =$$

$$16 + 8 + 3 = 27 \checkmark$$

$$230_4 = 2 \cdot 4^2 + 3 \cdot 4 + 0 = 32 + 12 = 44 \checkmark$$

Binary 6 bits

000000
000001
000010
000011
000100
000101
000110
000111
001000
001001
001010
001011
001100
001101
001110
001111
010000
010001
010010
010011
010100
010101
010110
010111
011000
011001
011010
011011
011100
011101
011110
011111
100000
100001
100010
100011
100100
100101
100110
100111
101000
101001
101010
101011
101100
101101
101110
101111
110000
110001
110010
110011
110100
110101
110110
110111
111000
111001
111010
111011
111100
111101
111110
111111

How many # we can represent?

How many possibilities in binary 6 bits? 64

→ Which 64 numbers do we represent? [0:63]

{0, 1, 2, ..., 63}

maybe {100: 163}

signed range [-32:31]
rep = two complement

{-32, -31, -30, ..., -1
0, 1, 2, ..., 31}

3 shirts S_A, S_B, S_C

2 pants P_1, P_2

4 hats H_A, H_B, H_C, H_D

Q: how many ways to dress up?

Product Rule

all choices can be combined
(every choice valid with any other

choice
- no restriction)

$$3 \times 2 \times 4 = 24 \text{ possib.}$$

$(S_A, P_1, H_A), (S_A, P_2, H_B) \dots$
 $(S_A, P_2, H_A), (S_A, P_2, H_B) \dots$

6 bits      

2^{pos} 2^{pos} 0/1 0/1 0/1 0/1
0/1 0/1

Product Rule? all choices are valid Yes

$$\# \text{ possib} = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^6.$$

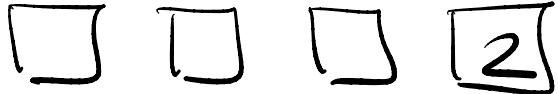
4 digits base 3    

$d \in \{0, 1, 2\}$ 3^{poss}
0/1/2 0/1/2 0/1/2 0/1/2

Product Rule? are all choices valid with any other choices? YES

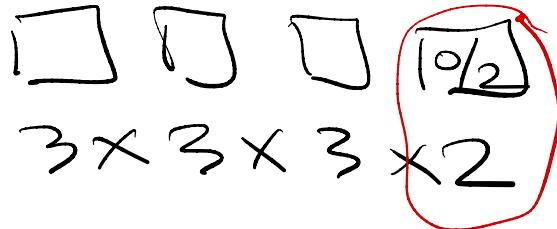
$$3 \times 3 \times 3 \times 3 = 3^4 = 81$$

how many possible base 3 4 digits
- ending with "2"



$$27 = 3 \times 3 \times 3 \times 1$$

- ending until "2" or "0"



$$3 \times 3 \times 3 \times 2$$

base 10 5 digits

dt {0, 1, 2, 3, 4, 5, 6, 7,
8, 9, 10}



$10 \times 10 \times 10 \times 10 \times 10$
 $\begin{matrix} 0 \\ \rightarrow \\ 9 \end{matrix} \quad \begin{matrix} 0 \\ \rightarrow \\ 9 \end{matrix}$

10^5 possibilities.

unsigned Range

{0, 1, 2, ..., 99999}

[0:99999]

Size of the range

$$|\text{Range}| \leftarrow 10^5 - 1$$

why?

$$99999 + 1 = \underline{\underline{100000}}_{10^5}$$

Range

What is max in Range (unsigned)

base 10 K digits.

$$\underline{d_{k+1}} \underline{\dots} \underline{\dots} \underline{\dots} \underline{d_3} \underline{d_2} \frac{d_1}{1} \frac{d_0}{0}$$

Generic
K+1

" $d_{k+1} d_{k+2} \dots d_1 d_0$ "

$$\text{power expansion: } d_{k+1} \cdot 10^{k-1} + d_{k+2} \cdot 10^{k-2} + \dots + d_1 \cdot 10^1 + d_0$$

$$\text{max possible: } 9 \cdot 10^{k-1} + 9 \cdot 10^{k-2} + \dots + 9 \cdot 10^1 + 9$$

$$= (10-1) (10^{k-1} + 10^{k-2} + \dots + 10^1 + 10^0)$$

$$10^k - 10^{k-1} +$$

$$10^{k-1} - 10^{k-2}$$

$$10^{k-2} - 10^{k-3}$$

$$\vdots \quad \vdots$$

$$10^1 - 10^0$$

telescope

$$10^k - 10^0 = 10^k - 1$$

Signed binary [6 bits] — — — — —

$2^6 = 64$ possibilities "E" in a set
 Range = [-32:31] belongs to

Mechanism/procedure to map each value [-32:31]
 to string of 6 bits Two's complement

positive
 use the other 5 bits for unsigned value

• Unsigned mechanism = stays.

$\begin{array}{r} 0 \\ \underline{\quad} \\ 0 \end{array}$ $\begin{array}{r} 0 \\ \underline{\quad} \\ 1 \end{array}$ $\begin{array}{r} 1 \\ \underline{\quad} \\ 0 \end{array}$ $\begin{array}{r} 0 \\ \underline{\quad} \\ 1 \end{array}$ $\begin{array}{r} 1 \\ \underline{\quad} \\ 1 \end{array}$ = 3 ✓

-1 = ? negative \Rightarrow first bit = 1
 derived value

$1 \underline{\quad} \underline{\quad} \underline{\quad} \underline{\quad} \underline{\quad} \underline{\quad} ?$

"Complement of 1" = ?
 $1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 0 \quad ?$ Not sure

$$\begin{array}{r}
 -1 = ? \\
 +1 = \boxed{1} \quad \underline{1} \quad \underline{1} \quad \underline{1} \quad \underline{1} \quad \underline{1} \quad \underline{?1} \\
 \hline
 0 = \boxed{0} \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0
 \end{array}$$

$$\begin{array}{r}
 -2 = ? \quad \underline{1} \quad \underline{1} \quad \underline{1} \quad \underline{1}^{(1)} \quad \underline{1} \quad \underline{?0} \\
 +2 = \quad 0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 0 \\
 \hline
 0 = \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0
 \end{array}$$

+32 (min in range) = ?

$\underline{1}$	$\underline{0}$	$\underline{0}$	$\underline{0}$	$\underline{0}$	$\underline{0}$	$\underline{0}$
-----------------	-----------------	-----------------	-----------------	-----------------	-----------------	-----------------

$$\begin{array}{r}
 +32 \\
 \text{unsigned} \\
 (\text{not sign bit}) \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0
 \end{array}$$

Exercise two's complement works like this

$$\begin{array}{cccccc} 1 & 0 & 1 & 1 & 1 & 0 \end{array}$$

Power expansion $= 1 \cdot 2^5 + 0 \cdot 2^4 + 1 \cdot 2^3 + 1 \cdot 2^2 + 1 \cdot 2^1 + 0 \cdot 2^0$

has the first power with negative sign.

$$\begin{array}{r} -9 = ? \quad | \quad | \quad 0 \quad | \quad | \quad 1 \\ +9 = 0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 1 \\ \hline 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \end{array}$$

2's complement
power exp

$$\begin{aligned} & \underline{\underline{1 \ 0 \ 1 \ 1 \ 1}} = \\ & -1 \cdot 2^5 + 1 \cdot 2^4 + 0 \cdot 2^3 + 1 \cdot 2^2 + 1 \cdot 2^1 + 1 \\ & -32 + \underline{\underline{16 + 4 + 2 + 1}} \\ & = -32 + 23 = -9 \end{aligned}$$