Theorem 1: Divisibility with 3 in base 10. A natural number is divisible with 3 if and only if the sum of its base10-digits is divisible with 3.

proof. Lets say $n = d_0 + d_1 * 10^1 + d_2 * 10^2 + ... + d_k * 10^k$ in base 10, with digits $d_i \in \{0, 1, 2..., 9\}$. That is the same as writing $n = \sum_{i=0}^{k} d_i 10^i$

Recall that modulo 3 means remainder at division with 3. For example 100 mod 3 = 1, because 100=3*33+1.
Also recall that modulo distributes over sum, product, and powers. For example
(100+22) mod 3 = ((100 mod 3) + (22 mod 3)) mod 3.
See the appendix for a recap of modulo operations.

So now we can write:
$n \mod 3 = (\sum_{i=0}^{k} d_i 10^i) \mod 3$
$= (\sum_{i=0}^{k} (d_i 10^i \mod 3)) \mod 3$
$= (\sum_{i=0}^{k} (d_i \mod 3) * (10^i \mod 3)) \mod 3$
$= (\sum_{i=0}^{k} (d_i \mod 3) * (10 \mod 3)^i) \mod 3$
$= (\sum_{i=0}^{k} (d_i \mod 3) * 1^i) \mod 3$
$= (\sum_{i=0}^{k} (d_i \mod 3)) \mod 3$
$= (\sum_{i=0}^{k} d_i) \mod 3$

Reading the beginning and the end in english : The remainder of n divided by 3 is the same as the reminder of the sum-of-digits(n) divided by 3. In particular if one of these remainder is 0 (that means divisible with 3) the other one is aslo 0.
This is a proof in both directions since we didnt use implications (unidirectional), we used equality modulo 3, which goes both ways.