Content-based Page Sharing with Universal and Perfect Hashing

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Background

- In recent years, the speed and capacity gap between processor and memory continues to widen
- Methods for efficient usage of space resource are becoming increasingly important
- Rather than pursuing extreme time efficiency as the previous decade

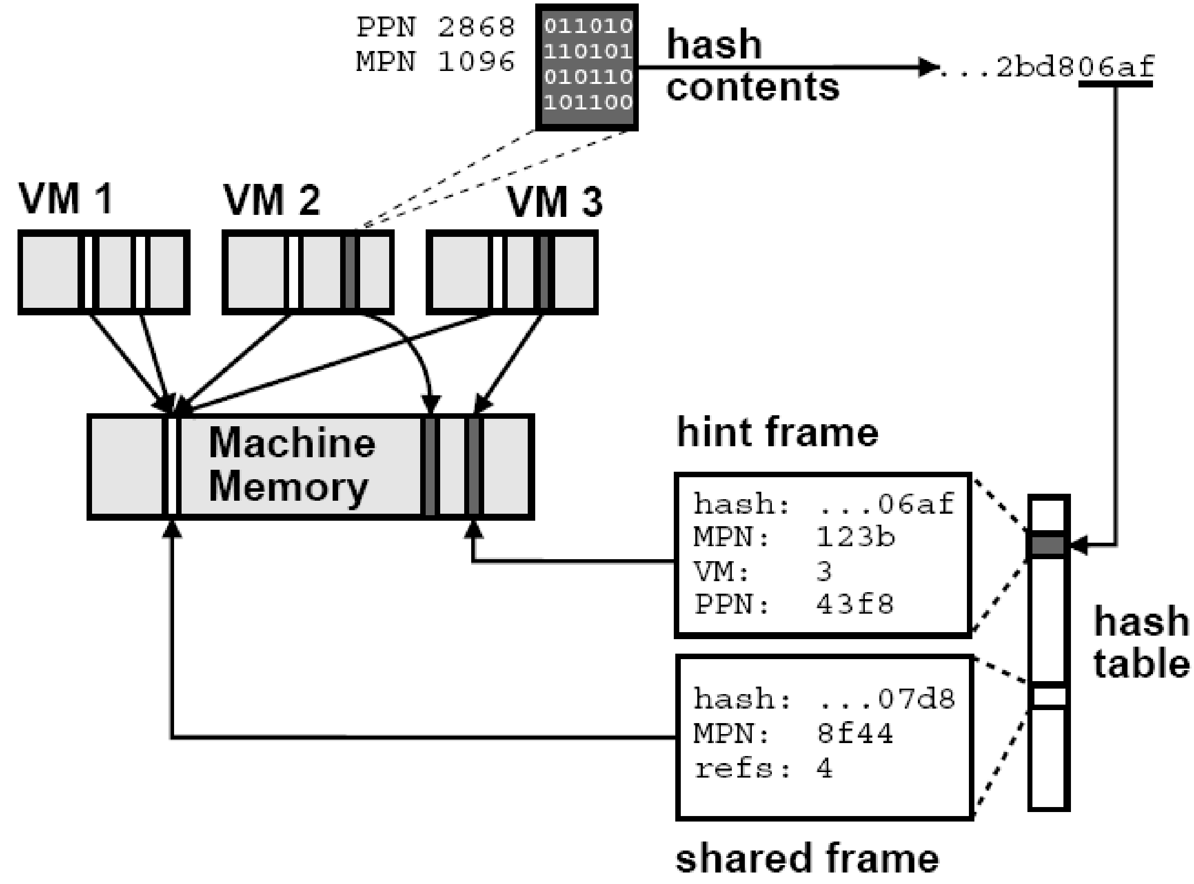
Multiple Virtual Machines

- Case: running multiple virtual machine instances on one single workstation
- Want: share memory across these VM instances to improve the memory usage efficiency.
- Observe: many pages are identical across the VMs, such as system kernel, device drivers, TCP/IP stack and etc.

VMware ESX Server

- VMware ESX Server allocates one single memory space for each instance.
- Each memory page in such spaces has its own **PPN** (Physical Page Number).
- VMware ESX Server maps these pages into the memory of the host machine, which maps the PPN to **MPN**(Machine Page Number).

Content-based page sharing



Key problem: Hash function

- Key problem: to construct a hash function mapping the length L long identifier to hash value of O(log L) length.
- Modern memory pages are usually 4K to 512K bits, which is a very long input for the hash function.
- Moreover we want the description length of this hash function should also be O(log L).

Hash Basics

- A universe U with some subset N∈U. We want to store the subset N using as little space as possible
- This function h: U -> {1,2,......M} is defined as the hash function.

- Definition: A collision occurs when h(x) = h(y) for two distinct keys x, y.

Hash function

- Claim: Let F be a hash function, that maps n elements to table [m], with proper m, then the expected number of collisions will be at most ½

- Proof:

$$Pr\,[F\ is\ not\ 1-1] = \Pr[\exists\, i \neq j: F(E_i) = F(E_j)]$$

$$= \Pr[F(E_1)=F(E_2)\ \text{or}\ F(E_2)=F(E_3)\ \text{or}\ldots\ldots]$$

$$\leq \binom{n}{2} Pr[F(E_1)=F(E_2)]$$

$$= \binom{n}{2}\frac{1}{m}$$

$$= \frac{n(n-1)}{2}\frac{1}{n^2} \leq \frac{1}{2}$$

Universal Hashing

- A **universal hash function** is one in which the probability of a collision between any two keys is provably 1/M.

- <u>Definition</u>: A randomized algorithm H for constructing hash function h: U -> {1,2,……M} is *universal* if for all x<>y in U, we have

$$Pr[h(x) = h(y)] \leq \frac{1}{M}$$

Recall: Finite Fields

- A finite field F is a set of objects with operations + and * that behave as you would expect as real space.
- Example: In a field F={0,1,2......12} with operation +, * and mod 13:

$$12 + 2 = 1$$
$$3 * 5 = 3$$
$$1/2 = 7$$

- Observe: For every prime P the above field with mod P is a finite field.

Polynomial over Finite Fields

- A polynomial over finite fields is an expression of the form

$$\sum_{i=1}^{N} \left(a_i * x^{i-1}\right) \ mod\ P$$

- For some non-negative integer n and where the coefficients are drawn from some designated set S.
- S is called the coefficient set. When a≠0, we have a polynomial of degree n.

Polynomial arithmetic

- We can add two polynomials:

$$f(x) = a_2x^2 + a_1x + a_0 \ mod\ P$$
$$g(x) = b_1x + b_0 \ mod\ P$$
$$f(x) + g(x) = a_2x^2 + (a_1 + b_1)x + (a_0 + b_0) \ mod\ P$$

- We can multiply two polynomials:

$$f(x) = a_2x^2 + a_1x + a_0 \ mod\ P$$
$$g(x) = b_1x + b_0 \ mod\ P$$
$$f(x) * g(x) = a_2b_1x^3 + (a_2b_0 + a_1b_1)x^2 + (a_1b_0 + a_0b_1)x + a_0b_0 \ mod\ P$$

- Theorem: Non-zero polynomial P(x) over a finite field F, with degree d, has at most d roots.

Hash function for long identifiers

- Find a prime P slightly larger than L^2
- Using modular P to define a hash function like following:

$$H_x(ID) \triangleq P_{ID}(x) = \sum_{i=1}^{N} \left(ID[i] * x^{i-1} \right) \, mod \; P$$

- Where ID[i] denotes the i-th bit of long identifier ID, and x is picked at random in [1..P].
- Represent the identifier as a polynomial over a finite field with modular P.

Proof: Universal Hashing

- Claim: For any two distinct IDs, the probability, over the choice of the hash function that their hashes coincide is at most 1/L.

- Approximately consider that this hash function hashes L-bit ID to a number in [1..L^2] ,which is of length 2*log(L)

Proof (cont’)

- For any two different ID and ID’, both of length L, will show that the probability that = can be induced to the probability taken over random x on . The equation $*$:

$$H_x(ID) - H_x(ID') = \sum_{i=1}^{L} \left((ID[i] - ID'[i]) * x^{i-1} \right) mod\ P$$

- Assume ID and ID’ have s bits common from the beginning, it will be a polynomial mod P of degree L-1-s.

Proof (cont')

- So $Pr[H_x(ID) - H_x(ID') = 0]$ will be the number of x's roots in the equation $*$ over the size of original ID set.
- (Note: P is a prime and we showed polynomial over a finite field with operation modular P has most degree d roots)

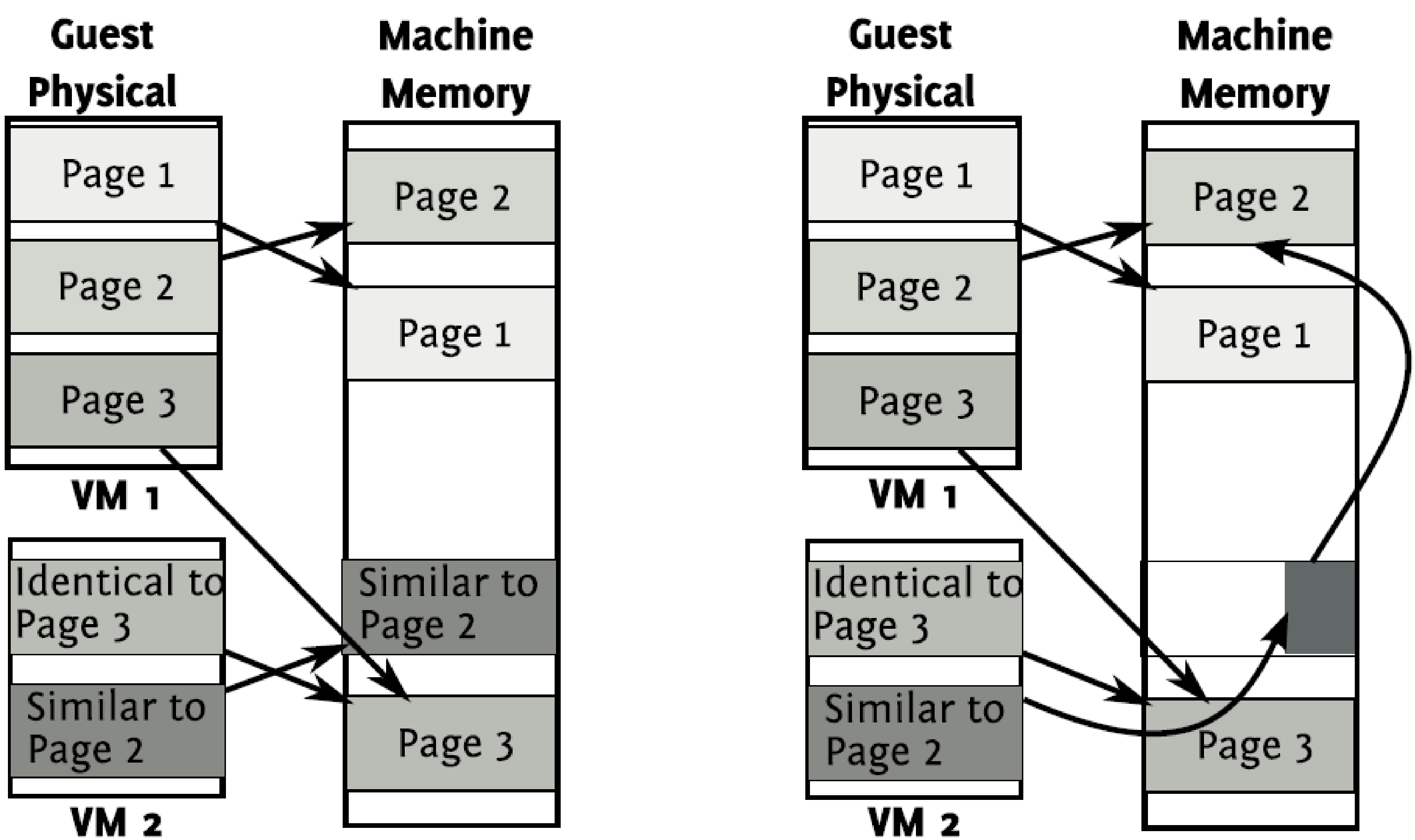
$$Pr[H_x(ID) - H_x(ID') = 0] = \frac{L-1-s}{P} < \frac{L-1}{L^2} < \frac{L}{L^2} = \frac{1}{L}$$

- Thus we prove the collision is at most 1/L.

Perfect Hashing

- The above construction is the first step of my hash function construction. What we want next is **Perfect Hashing**, with zero collision.
- Idea: two level hashing. For all the buckets with collision, we generate another hash function which will give no collisions for the items in such buckets.
- Still bound the total space of the hash function.

Future Work


Guest Physical
Machine Memory
Page 1
Page 2
Page 3
VM 1
Identical to Page 3
Similar to Page 2
VM 2
Page 2
Page 1
Similar to Page 2
Page 3
Guest Physical
Machine Memory
Page 1
Page 2
Page 3
VM 1
Identical to Page 3
Similar to Page 2
VM 2
Page 2
Page 1
Page 3

Thank You!