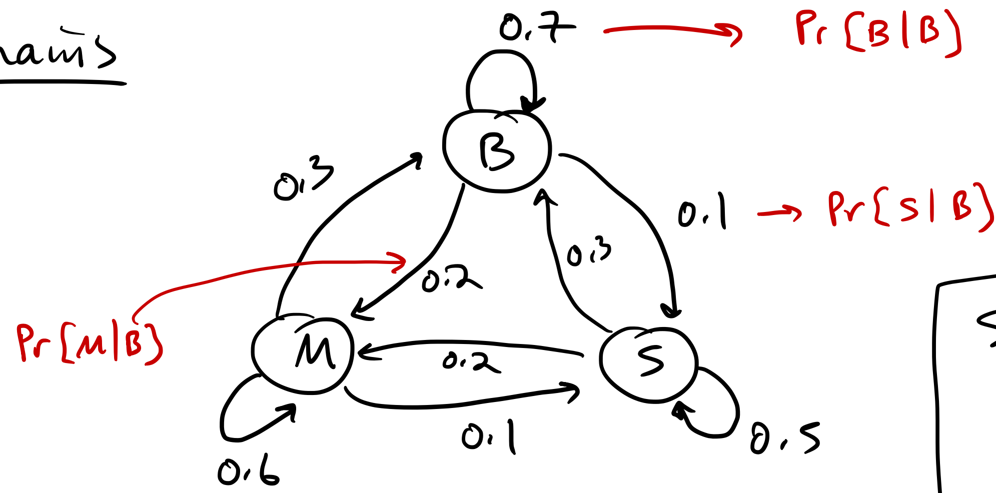
Markov chains

B: Bertucci's

M: Margaritas

S: Sato



3 reds ⇒ must add to 1

state Transition Matrix

$$P = \begin{array}{c|ccc} & B & M & S \\ \hline B & .7 & .2 & .1 \\ M & .3 & .6 & .1 \\ S & .3 & .2 & .5 \end{array}$$

Stochastic matrix

⇒ all rows sum to 1

$$B = 0.7 \cdot B + 0.3 \cdot M + 0.3 \cdot S$$

Stationary Distribution: long-term fraction of time spent visiting each state.

B, M, S

$$\vec{\pi} = \langle \pi_B, \pi_M, \pi_S \rangle$$

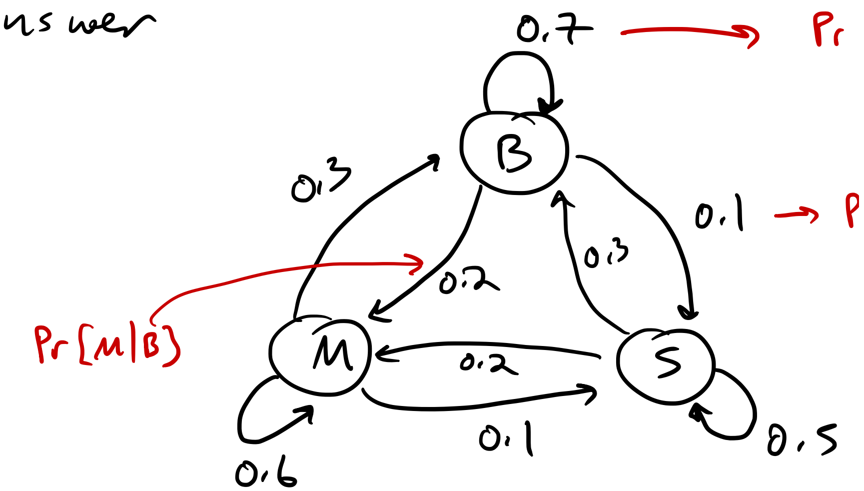
$$\pi_B + \pi_M + \pi_S = 1$$

Finding the s.d.:

① Simulation

– highly ~~inefficient~~

(2) Create equations & solve for exact answer



(1) $B = .7B + .3M + .3S$

(2) $M = .2B + .6M + .2S$

(3) $S = .1B + .1M + .5S$

But... I have one more equation:

(1) $.3B - .3M - .3S = 0$

(2) + (3) $.3B - .3M - .3S = 0$

(4) $B + M + S = 1$

$$\left.\begin{aligned} B + M + S &= 1 \\ M &= .2B + .6M + .2S \\ S &= .1B + .1M + .5S \end{aligned}\right) \Rightarrow \begin{aligned} B + M + S &= 1 && (1) \\ -.2B + .4M - .2S &= 0 && (2) \\ -.1B - .1M + .5S &= 0 && (3) \end{aligned}$$

(1) $B + M + S = 1$

$5\times$(2) $-B + 2M - S = 0$ $\Rightarrow$

$10\times$(3) $-B - M + 5S = 0$

$B + M + S = 1$

$3M = 1 \Rightarrow M = 1/3$

$6S = 1 \Rightarrow S = 1/6$

$\Rightarrow B = 1/2$