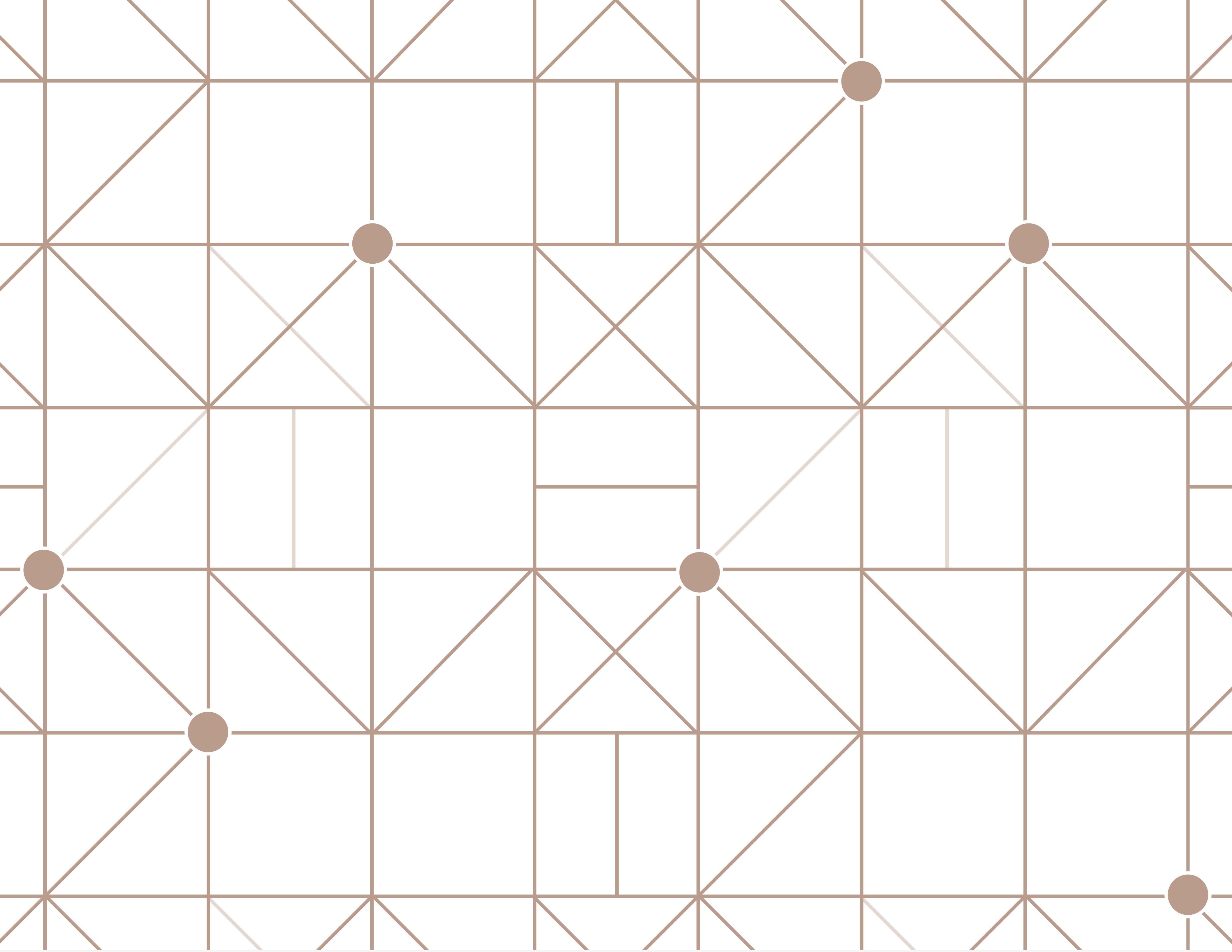

OH 4/11/2021 AA 17.3-3.

n_i = # nodes in heap D_i after i operations

$\Phi(D_i) = K n_i \ln(n_i)$ → natural log

K = large enough const based on heap ops ins, ext-min

op $= \Theta(\log n) \Rightarrow K$ Θ const

$\forall$ ops $\leq K \cdot \log(n)$ $\forall n$

TRICK $\boxed{n \cdot \ln\left(\frac{n}{n-1}\right)} = \Theta(1)$

$\Leftrightarrow n \cdot \ln\left(\frac{n}{n-1}\right) \leq$ const c

$\Leftrightarrow \lim_{n\to\infty} n \ln\left(\frac{n}{n-1}\right) =$ finite const

proof: $\lim_{n} n \ln \frac{n}{n-1} = \lim_{n\to\infty} \ln\left[\left(1+\frac{1}{n-1}\right)^n\right] =$

$= \lim_{n\to\infty} \ln\left[\underbrace{\left(1+\frac{1}{n-1}\right)^{n-1}}_{\lim_{n}() = e}\right]^{n/n-1} = \lim_{n\to\infty} \ln(e)^{n/n-1} =$

$= \lim 1^{n/n-1} = 1$

INSERT $\hat{c}_i = c_i + \Phi(D_i) - \Phi(D_{i-1})$

$n_{i-1} = n_i - 1$

$c_i \le k\ln(n_i)$ true cost upper bound

$$\le k\ln(n_i) + kn_i\ln(n_i) - kn_{i-1}\ln(n_{i-1})$$

$$= k\ln(n_i) + kn_i\ln(n_i) - k(n_i-1)\ln(n_i-1)$$

exclude

$$\cancel{\le k\ln(n_i) + kn_i\ln(n_i) - kn_i\ln(n_i-1)}$$

$$= k\ln(n_i) + kn_i\ln(n_i) - kn_i\ln(n_i-1) + k\ln(n_i-1)$$

$$\le 2k\ln(n_i) + kn_i\left[\ln(n_i) - \ln(n_i-1)\right]$$

$$= 2k\ln(n_i) + kn_i\ln\frac{n_i}{n_i-1}$$

TRICK $\le$ const c

$$\le 2k\ln(n_i) + k\cdot c$$

k, c constants

$$= O(\log(n_i))$$

$\boxed{\text{Extract Min}}$ $\hat{c}_i = c_i + \Phi(D_i) - \Phi(D_{i-1})$

$\leq k \ln(n_0)$

$n_i = n_{i-1} - 1$

$$\leq k\ln(n_{i-1}) + k n_i \ln(n_i) - k n_{i-1} \ln(n_{i-1})$$

$$= k\ln(n_{i-1}) + k n_i \ln(n_{i-1} - 1) - k n_{i-1} \ln(n_{i-1})$$

$(n_{i-1} - 1)$

$$= k\ln(n_{i-1}) + k n_{i-1} \ln(n_{i-1} - 1) - k\ln(n_{i-1} - 1) - k n_{i-1} \ln(n_{i-1})$$

$$= k\underbrace{\left[\ln(n_{i-1}) - \ln(n_{i-1} - 1)\right]} + \cancel{k}\, n_{i-1} \underbrace{\left[\ln(n_{i-1} - 1) - \ln(n_{i-1})\right]}_{\text{trick}}$$

$$= \ldots \ln\left(\frac{n_{i-1}}{n_{i-1} - 1}\right)$$

easy

$$= \ldots$$

$$= O(1)$$

Easier $\Phi(D_i)$:
(hard to come up with)

x = node/value in heap

$\text{depth}_i(x)$ = depth/level of x after i-th operation.

$$\Phi(D_i) = \sum_{x \in \text{heap}} k(\text{depth}(x) + 1)$$

because each op only affects one value (x)