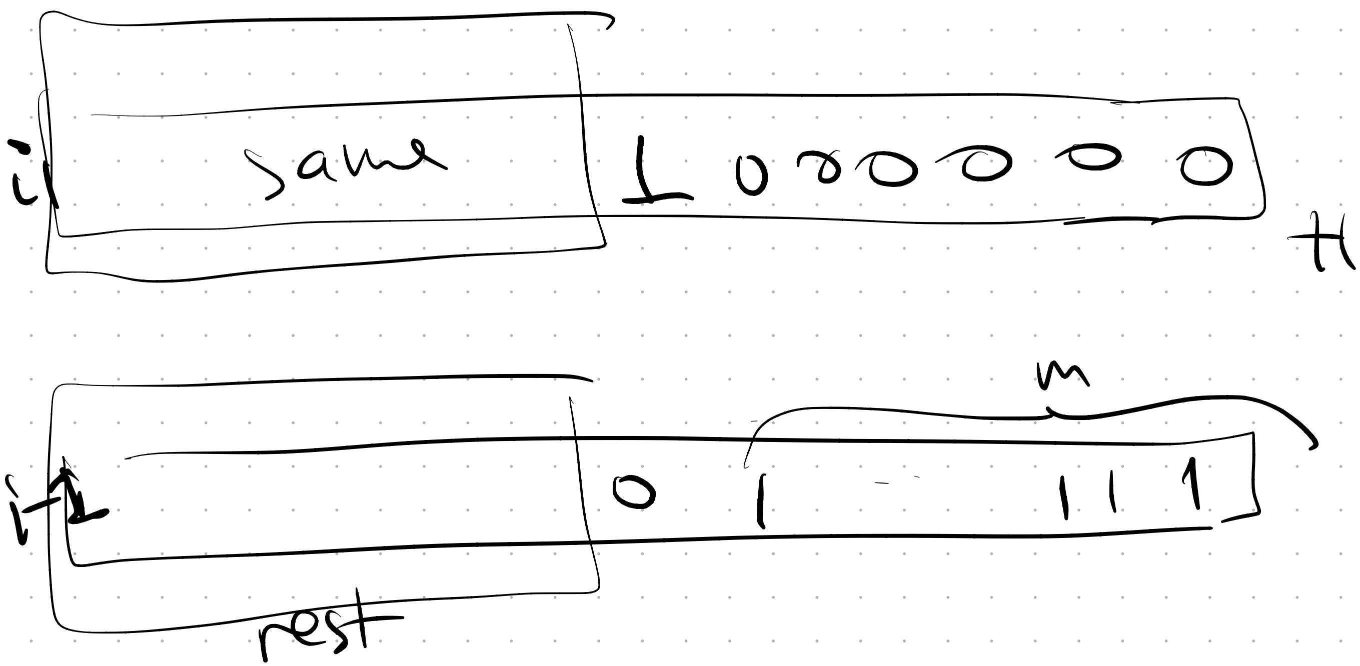
true cost = $m+1$

$$\boxed{\hat{c}_i = c_i + \Delta\Phi_i = m+1 + (1-m)} = 2$$

$$\Delta\Phi = 1-m$$



$$\Phi = 1 + \Phi(\text{rest})$$

$$\Phi = m + \Phi(\text{rest})$$

Amortized Analysis

n ops $\Rightarrow$ counter $= n$

binary counter 6 bits operation: $+1$ cost = # bits changed

global # changes in n ops $\frac{n}{16} + \frac{n}{8} + \frac{n}{4} + \frac{n}{2} + n \leq 2n$

global count method

$\Phi(T_i) =$ # of "1" bits

b_5	b_4	b_3	b_2	b_1	b_0	true cost	$\Phi(T_i)$	$\hat{c}_i = c_i + \Delta\Phi_i$
0	0	1	0	0	0	4	1	2
0	0	0	1	1	1	1	3	2
0	0	0	1	1	0	2	2	2
0	0	0	1	0	1	1	2	2
0	0	0	1	0	0	3	1	2
0	0	0	0	1	1	1	2	2
0	0	0	0	1	0	2	1	2
0	0	0	0	0	1	1	1	2
0	0	0	0	0	0		0	2

naive analysis: n operations $\times \underbrace{\Theta(\log n)}_{\text{worst case per op}} = \Theta(n \log n)$

Stack operations

push(x) → push x to top

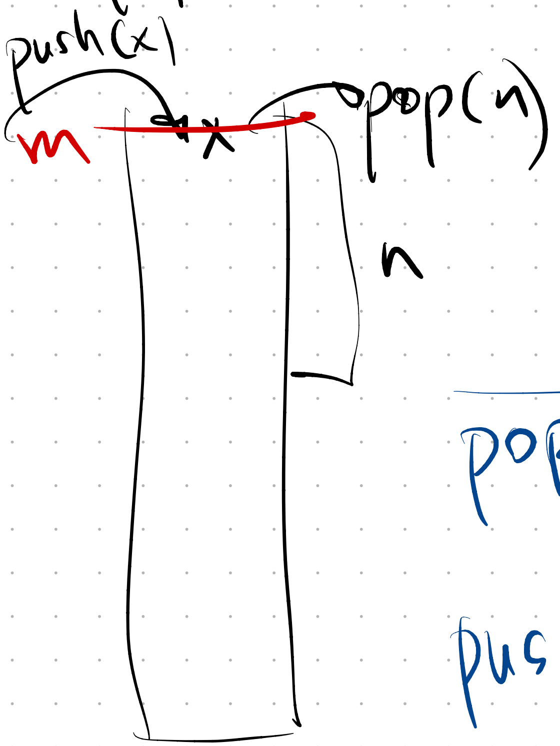
pop(n) → pops n value from top (or all if ≤ n avail)

n operations

- valid stack

naive run time

n × worst case pop(n) $= \Theta(n^2)$



- accounting method

	true cost c_i	amortized cost $\hat{c}_i$	(savings)
pop(n)	n	0	(−n)
push	1	2	(+1)

c_i = true cost of op_i

$\hat{c}_i$ = amortized cost of op_i

∀k

k-ops

Guarantee: $\sum_{i=1}^{k} c_i \le \sum_{i=1}^{k} \hat{c}_i$

what we payed so far (k) is enough to cover actual exp.

$\phi(\text{stack}) = \text{size of stack} = m$ $\quad \Delta$

$$\widehat{push} = push + \Delta\phi_{push} = 1 + 1 = 2$$

k lower

$$\widehat{pop(k)} = pop(k) + \Delta\phi_{pop} = k + (-k) = 0$$

- potential (situation, danger, $\boxed{\text{datastructure}}$)

T_i = datastructure / situation after i-th operation

$\phi(T_i) \geq 0 \qquad \phi(T_0) = 0$

suf cond $\quad \hat{c}_i - c_i \geq \Delta\phi_i$

$$\boxed{\hat{c}_i \geq c_i + \underbrace{\phi(T_i) - \phi(T_{i-1})}}$$

$\Delta\phi_i$ = change in potential due to op i

Th: suf cond happens $\forall i \Rightarrow$ Guarantee works.

proof: $\sum_{i=1}^{k} \hat{c}_i \geq \sum_{i=1}^{k} c_i + \phi(T_i) - \phi(T_{i-1}) = \sum_{i=1}^{k} c_i +$

$\phi(T_k) - \cancel{\phi(T_{k-1})} +$

$\cancel{\phi(T_{k-1})} - \cancel{\phi(T_{k-2})} +$

$\cancel{-} \qquad \cancel{-}$

$\cancel{\phi(T_1)} - \phi(T_0)$

$$= \sum_{i=1}^{k} c_i + \underbrace{\phi(T_k) - \phi(T_0)}_{\geq 0}$$