

OH Sat 10/9 5pm.

16.2-6 How to do it without sorting by quality

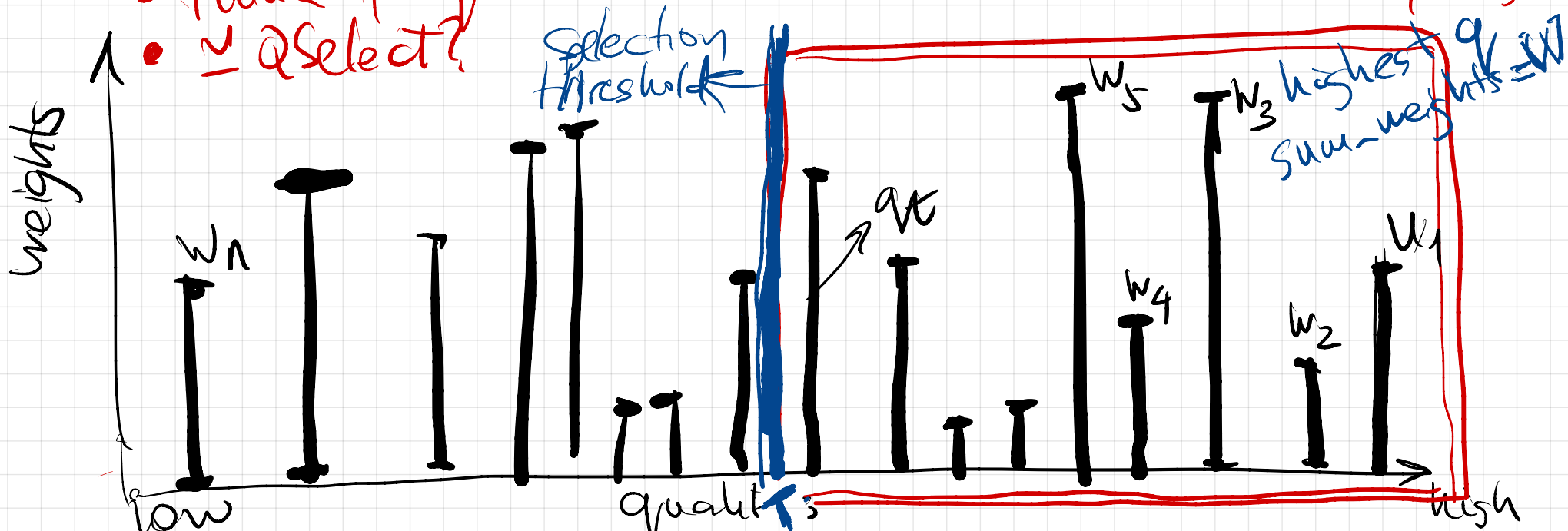
$$q_i = \frac{\text{val}_i}{\text{weight}_i}$$

$q_1 \geq q_2 \geq \dots \geq q_n$ → rational/real values

pick highest qual avail
or knapsack full.
repeat

HINT

- create array of qualities q_i not sorted.
- think of qualities in sorted order cumulative by weight
- ~ QSelect?



We want quality threshold T

$q_i \geq T$ are selected
 $q_i < T$ not selected

such that $t \in \mathbb{Z}$
→ index t of quality that
 $q_t = T$

\sum selected quality-items = W

use Q_{selected} modified?

$\mathbb{R} \ni W$ acts kind of like a rank for cumulative weights

on q -cumul array

• cumulative weights

$$\overline{w_i} = \sum_{j=1}^i w_j$$

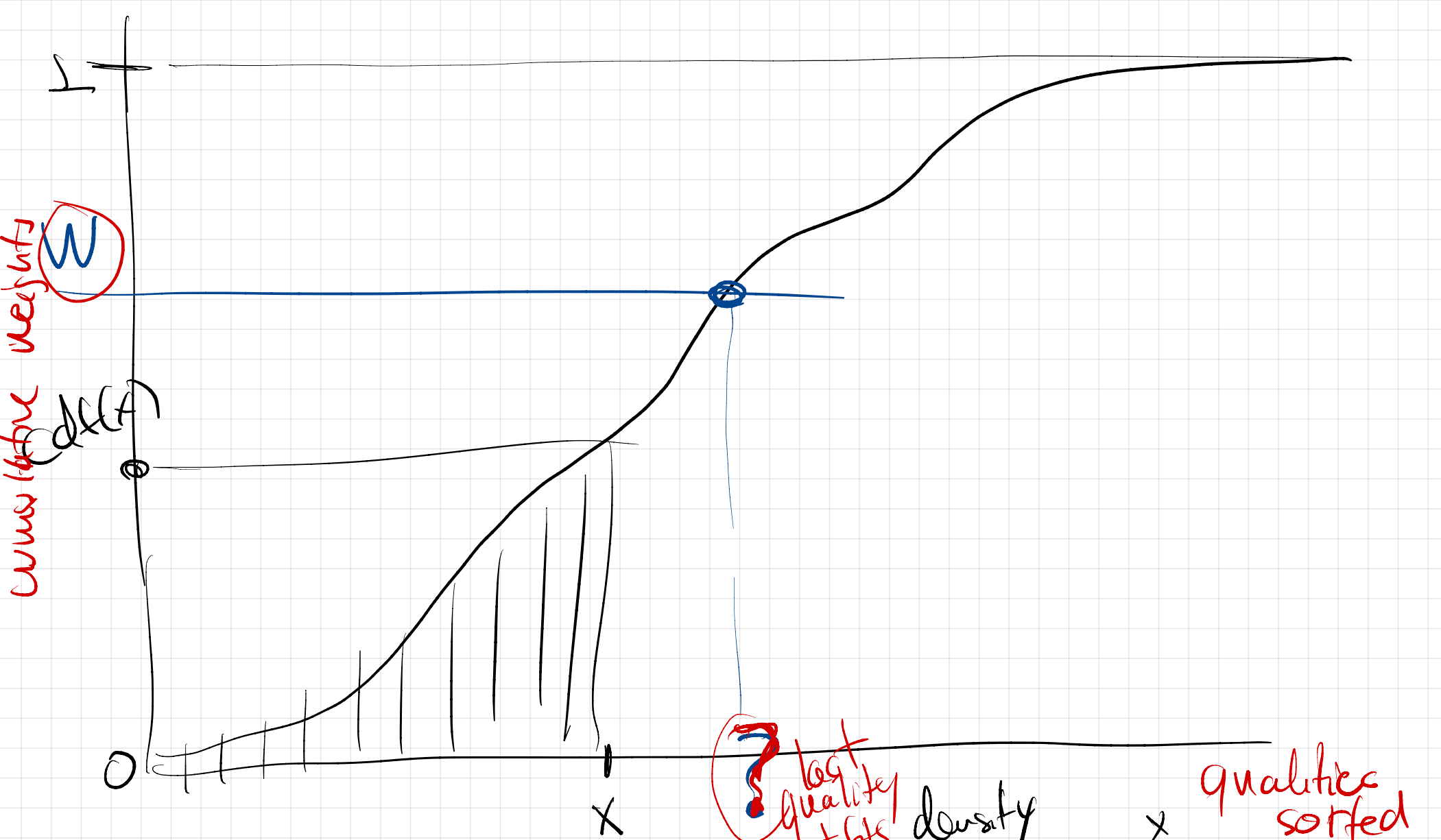
$\overline{w_1}, \overline{w_1 + w_2}, \overline{w_1 + w_2 + w_3}$

selected items

$\overline{w_1 + w_2 + \dots + w_n}$

$W = knapsack$ total weight

Cumulative distrib function



cumulative weights

W

$cdf(x)$

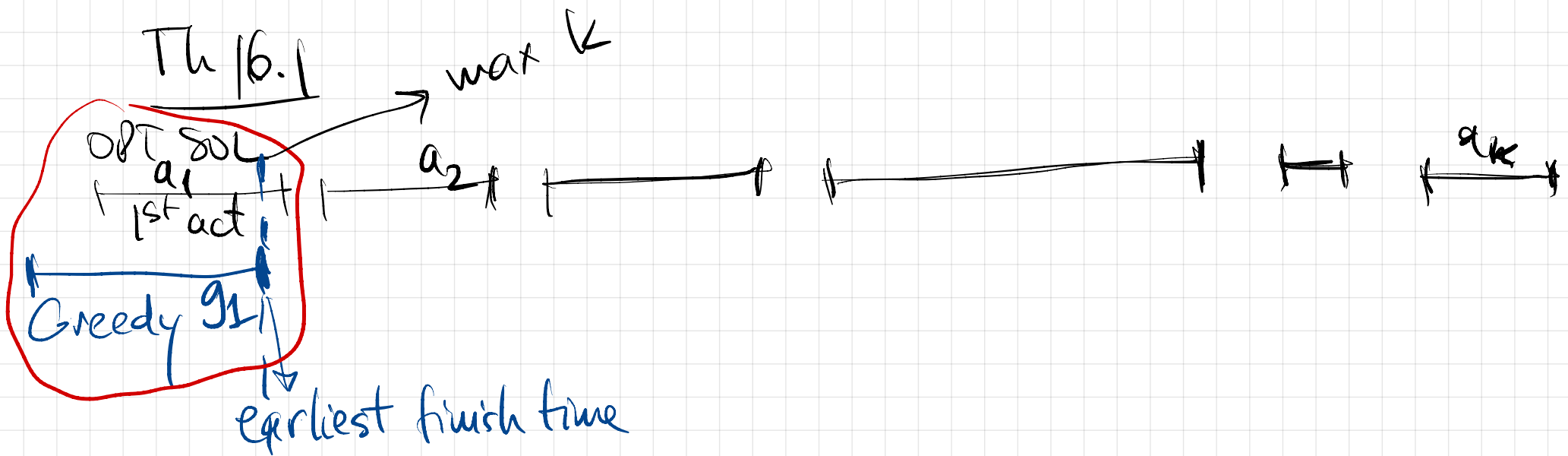
? best quality that fits

$$cdf(x) = \sum_{i \leq x} prob(i)$$

density

qualities sorted

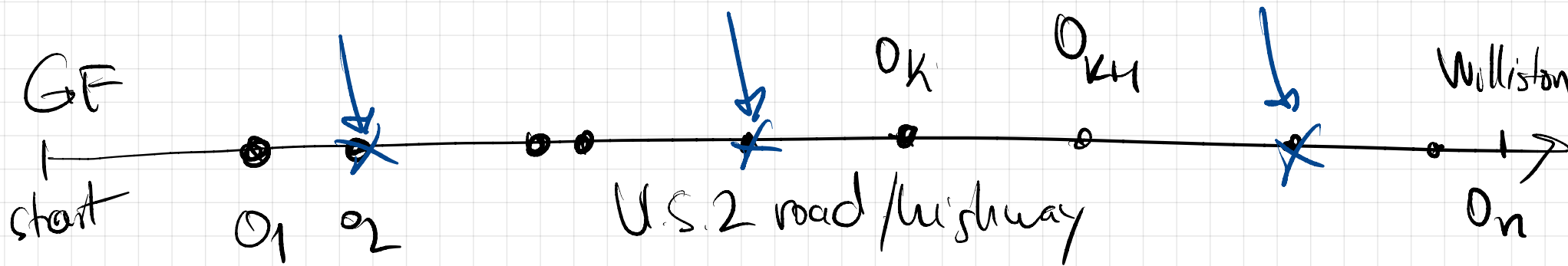
$$cdf(x) = \int_{t=0}^x prob(t) dt$$



exchange
 OPT SOL: replace its first activity a_1 with the greedy choice g_1
 \Rightarrow still optimal solution \Rightarrow Greedy works

$OBS = k$
still valid (no overlap)

16.2-4



map \Rightarrow Sorted in direction of travel

$|O_{k+1} - O_k| < \boxed{m}$ miles to go on bottled water.

$\Theta(n)$ time if locations are sorted

= frog: jump as far as possible (works)
in class
- have to output the location used

16-2-3.

$$w_1 \leq w_2 \leq w_3 \dots \leq w_n$$

$$v_1 \geq v_2 \geq v_3 \dots \geq v_n$$

$$q_1 \geq q_2 \dots \geq q_n$$

$Z =$ knapsack total weight

ind step: if the first k greedy choices are part of OPT SOL

items selected so far

$1, 2, \dots, k$ in order

$$w_1 + w_2 + \dots + w_k \leq Z$$

OPT SOL contains items $1:k$

the next greedy selection $k+1$ is also part of OPT SOL

$1, 2, \dots, k$ items from before

add item $k+1$

if w_{k+1} fits while still OPT SOL.

~~or fill knapsack~~

16-2-5

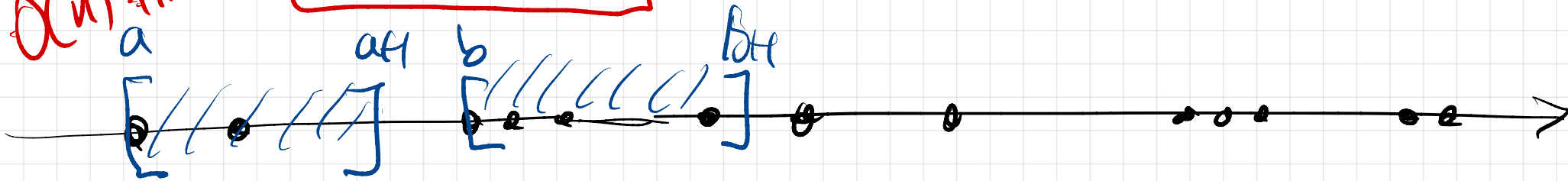
$\{x_1, x_2, \dots, x_n\}$ points on real line.

assume sorted
 $O(n)$ time

Sorted?
 $x_1 \leq x_2 \leq x_3 \leq \dots \leq x_n$

Not-sorted?

what then $O(n \log n)$



Want coverage with unit intervals $[a, a+1]$

16-1 a) US denom = $\{1, 5, 10, 25\}$ Greedy = biggest coin.
cents Proof: $n \geq 25 \Rightarrow$ add $25 \in \text{OPT SOL}$

$25 > n \geq 10 \Rightarrow$ add $10 \in \text{OPT SOL}$

$10 > n \geq 5 \Rightarrow$ add $5 \in \text{OPT SOL}$

b) Same proof denom = $\{c^0=1, c, c^2, \dots, c^k\}$

Proof
 $\forall t$ $c^t > n \geq c^t \Rightarrow$ add $c^t \in \text{OPT SOL}$

ex $n = 8792$

coins denom = $\{10^0, 10^1, 10^2, 10^3, \dots\}$ $c=10$
 $8 \cdot 10^3 + 7 \cdot 10^2 + 9 \cdot 10^1 + 2 \cdot 10^0$