System Specification, Verification and Synthesis (SSVS) – CS 4830/7485, Fall 2019

12: Formal Verification: Reachability

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Where we stand in the course

- Systems: DONE!
- Specification: Almost done! (we'll talk about automata later)
- Verification: next
- Synthesis: after that

Outline

- Verification
- Reachability analysis
- Counterexamples

VERIFICATION

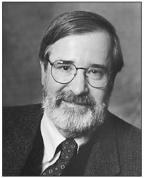
Verification and Computer-Aided Verification

- Systems: DONE!
- Specification: DONE (with temporal logics)!
- At this point, you should be able to do formal system modeling and specification.
- You could also in principle do verification "by hand", or using a general tool like a theorem-prover: plug in the definitions, try to prove the model-checking theorems.
- This is difficult to do by hand (theorem provers also typically require a lot of human interaction).
- So we turn to **computer-aided** and ideally **fully automated** verification.
- A.K.A. **model-checking**.

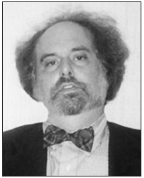
ACM Turing Award for Model-Checking

Clarke, Emerson, and Sifakis won the ACM Turing Award in 2007,

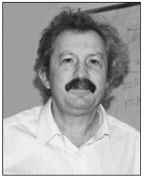
for their role in developing Model-Checking into a highly effective verification technology that is widely adopted in the hardware and software industries.



Edmund M. Clarke



E. Allen Emerson



Joseph Sifakis

Recall: the model-checking problems for LTL and CTL

Given:

- the implementation: a transition system (Kripke structure) $M = (\mathsf{AP}, S, S_0, L, R)$
- the specification: a temporal logic (LTL or CTL) formula ϕ

Check where M satisfies ϕ:

$$M \stackrel{?}{\models} \phi$$

- If ϕ is LTL: **every execution trace** of M must satisfy ϕ.
- If ϕ is CTL: **every initial state** of M must satisfy ϕ.

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For finite-state M, these questions can be answered fully automatically (problems are decidable)!

REACHABILITY ANALYSIS

Some model-checking problems are easier than others

For the same system M, some formulas may be easier to check than others.

Examples of two (conceptually) easy problems:

- checking deadlocks
- checking invariants

Checking Deadlock-Freedom

Checking that a system has no deadlocks (is **deadlock-free**) is conceptually easy:

- Explore (generate) all reachable states of the system.
- Check that none of them is a deadlock.[1]

[1] Some may be "legal end states", i.e., states without successors but which don't count as deadlocks because they have been identified (labeled) by the user as legal end states.

Recall: invariants

Suppose ϕ is of the form

$$\mathbf{G}\psi \qquad \text{or} \qquad \mathbf{AG}\psi$$

where ψ is a propositional formula (i.e., a boolean expression on atomic propositions).

E.g.,

$$\mathbf{G}(p \vee q), \qquad \mathbf{G}(p \rightarrow q), \qquad \cdots$$

Then ψ must be an **invariant**: it must hold at all reachable states.

Examples:

- "Whenever train is at intersection the gate must be lowered"
- "If the autopilot is off then the pilot must not believe it is on"

Model-Checking Invariants

Checking that ψ is an invariant is conceptually easy:

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Only infinite paths count for the verification of a property such as $\mathbf{G}p$. If the system deadlocks after every time it violates p, then, formally speaking, it satisfies $\mathbf{G}p$!

So, what to do?

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So, what to do? Check deadlock-freedom before you check invariants!

They both use the same method: **reachability analysis**!

Reachability analysis

So, both for deadlocks and invariants, we want to:

- Explore (generate) all reachable states: this is called **reachability analysis**.

Sometimes it's also called **state-space exploration**.

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So, both for deadlocks and invariants, we want to:

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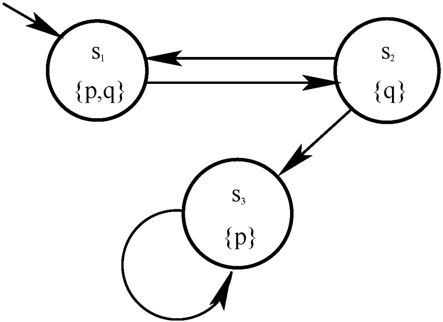
Sometimes it's also called **state-space exploration**.

- For finite-state systems, it can be done exhaustively and fully automatically!
- ... at least in theory ... in practice, often **state explosion** ...

Finite transition systems = Finite directed graphs



Any algorithm that explores all nodes of a graph can be used to explore all reachable states of a transition system!

Reachability analysis: summary

- Generate all reachable states ...
- ... while at the same time checking that each of them is "OK", i.e.,
    - it is not a deadlock state
    - it does not violate an invariant
    - ...

Reachability methods

- **Enumerative** (also called "explicit state").
  - These are basically search algorithms on directed graphs.

- **Symbolic** (we will see these later)
  - Bounded model-checking using SAT/SMT solvers.
  - Symbolic reachability.

ENUMERATIVE (EXPLICIT-STATE) REACHABILITY

Two standard search algorithms

- Depth-First Search (DFS)
- Breadth-First Search (BFS)

Depth-First Search

Assume given: Kripke structure (P, S, S_0, L, R).

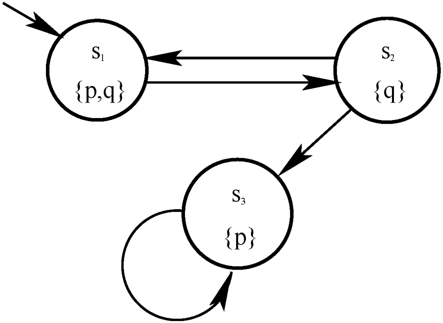
main:

```
V := ∅;                                      /* V: set of visited states */
for all s ∈ S0 do
    DFS(s);
end for
```

DFS(s):

```
check s;             /* is s a deadlock? is given p ∈ L(s)? ... */
V := V ∪ {s};
for all s′ such that (s, s′) ∈ R do
    if s′ ∉ V then
        DFS(s′);                             /* recursive call */
    end if
end for
```

Depth-First Search



Let's simulate DFS on this graph.

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Quiz:

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- Does it visit all reachable states? Yes: if s is reachable, then either $s \in S_0$, or s is the immediate successor of some s', which is itself reachable. In the first case, s is inserted into V because of the main loop. In the second case, assuming (by induction) that s' is inserted to V, s will also be inserted to V by loop in lines 3-6.
- Does it visit any unreachable states?

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- Does it visit any unreachable states? No: following the "inverse" of the argument above, if s is inserted into V, either this is done because of the main loop, or because of the loop in lines 3-6. In the first case, s must be in S_0, so it's an initial state, so it's reachable. In the second case, s must be successor of some s', which by induction must be itself in V, therefore reachable.
- What is the complexity of the algorithm?

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- What is the complexity of the algorithm? $O(n + m)$ where n is number of nodes/states and m is number of edges/transitions in the graph. Every node and edge are visited at most once.

Breadth-First Search

Assume given: Kripke structure (P, S, S_0, L, R).

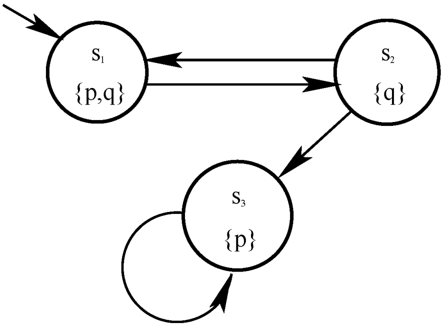
main:

FIFO queue $V := S_0$; /* V: queue of visited states */
set $E := \emptyset$; /* E: set of explored states */
BFS();

BFS:

while V non-empty **do**
  $s :=$ head(V);
  check s; /* is s a deadlock? is given $p \in L(s)$? ... */
  $E := E \cup \{s\}$;
  for all s' such that $(s, s') \in R$ and $s' \notin E \cup V$ **do**
    add s' to the end of queue V;
  end for
end while

Breadth-First Search



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- What is the complexity of the algorithm? $O(n + m)$ where n is number of nodes/states and m is number of edges/transitions in the graph. Every node and edge are visited at most once.

Other enumerative algorithms

Every search algorithm on finite graphs can be used for reachability analysis:

- Best-first search:
  - every state is assigned a “value” (using some heuristic value function, e.g., how “close” we are likely to be to the goal – in our case a “bad” state) and then next state to explore is the one with the highest value.
- A*: classic search technique in artificial intelligence.
- ...

But isn't the complexity of graph search awesome?!

$O(m + n)$ is a great complexity, right?
Not really...

- Most of these algorithms (DFS, BFS, Best-first, A*, ...) have been tried by researchers in verification.
- Basic complexity is the same for all: need to store all reachable states
  - in the "worst case" from the algorithmic point of view
  - but in fact "best case" from the verification point of view, since we are trying to prove that our system is correct! $\Rightarrow$ all reachable states must be correct
- **State explosion**: the number of reachable states is too large 

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So even reachability is a hard problem (both theoretically and in practice).

Enumerative methods to remedy state explosion

- **Bit-state hashing**: instead of storing the entire state vector, just store 1 bit per state: its hash value [Holzmann, 1998].
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  - And as we saw, even 1 bit per state may be too much already.
- **Partial-order reduction**: in asynchronous concurrent systems, transitions of different processes are often independent $\Rightarrow$ no need to explore all interleavings [Valmari, 1990, Godefroid and Wolper, 1991].
- **Symmetry reduction**: many state spaces are symmetric $\Rightarrow$ equivalence relation on states $\Rightarrow$ suffices to explore just one state per equivalence class, e.g., see [Sistla and Godefroid, 2004].
- ...

All these help, but don't eliminate the state-explosion problem.
Note: above references are representative, there is a lot more work on these topics.

STATE EXPLOSION in Spin and nuXmv

State explosion in Spin

```
// an illustration of state explosion
// as you increase N, the state space increases exponentially

#define N 7

    active [N] proctype p()     // N processes
    {
        l0: skip;
        l1: skip;
        l2: skip;
        l3: skip;
        l4: skip;
        l5: skip;
        l6: skip;
        l7: skip;
    }
// analysis:
// spin -run -noreduce state-explosion.pml
// spin -run state-explosion.pml
```

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