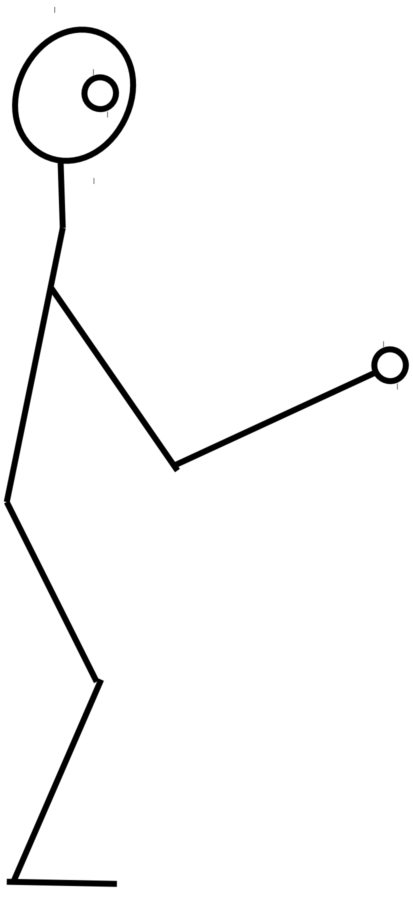
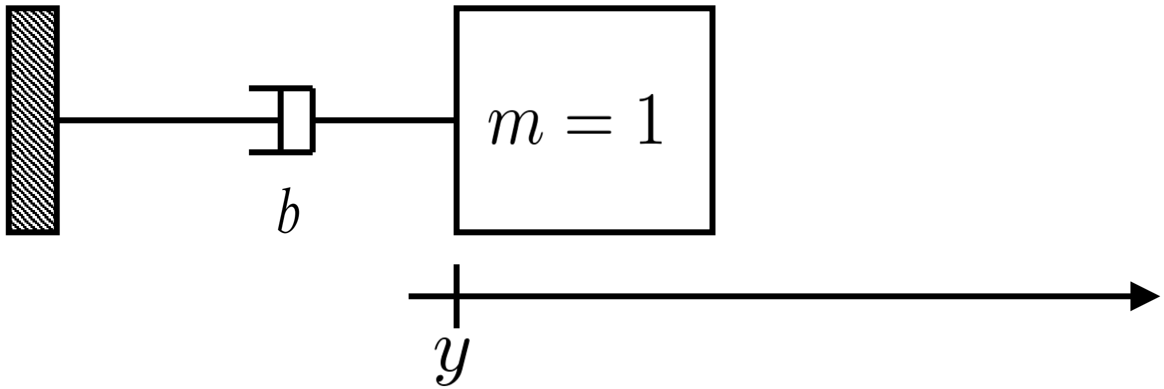
Linear Optimal Control



- What does optimal control mean?
- How could one possibly calculate such a thing?
- Does this work?

Double integrator



Equation of motion: $\ddot{y} + b\dot{y} = u$

Integrate forward one timestep:

$$y_{t+1} = y_t + \dot{y}dt$$
$$\dot{y}_{t+1} = \dot{y}_t + \ddot{y}dt$$

$$x = \begin{pmatrix} y \\ \dot{y} \end{pmatrix}$$

$$\begin{pmatrix} y_{t+1} \\ \dot{y}_{t+1} \end{pmatrix} = \begin{pmatrix} 1 & dt \\ 0 & 1 - bdt \end{pmatrix} \begin{pmatrix} y_t \\ \dot{y}_t \end{pmatrix} + \begin{pmatrix} 0 \\ dt \end{pmatrix} u_t$$

2D Double integrator

$$A = \begin{pmatrix} 1 & 0 & dt & 0 \\ 0 & 1 & 0 & dt \\ 0 & 0 & 1 - dtb & 0 \\ 0 & 0 & 0 & 1 - dtb \end{pmatrix}$$

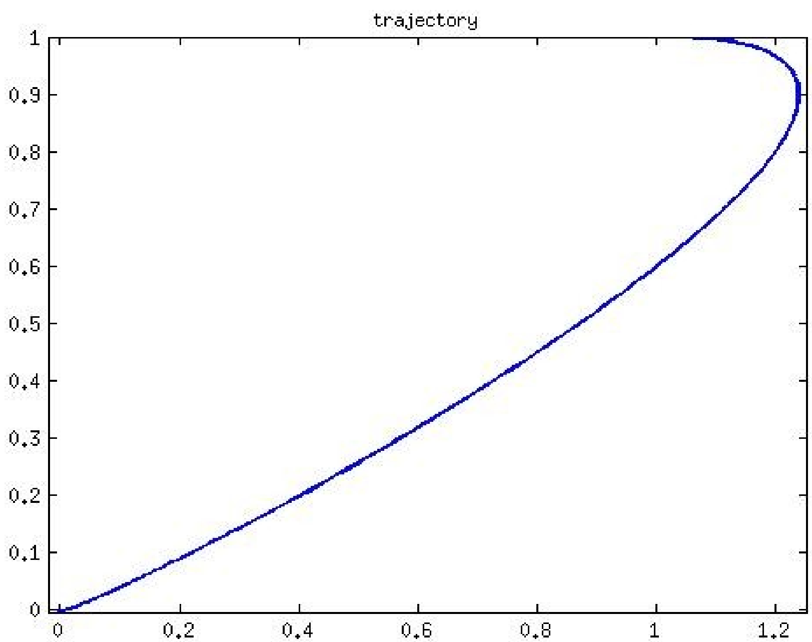
$$x = \begin{pmatrix} x_1 \\ x_2 \\ \dot{x}_1 \\ \dot{x}_2 \end{pmatrix}$$

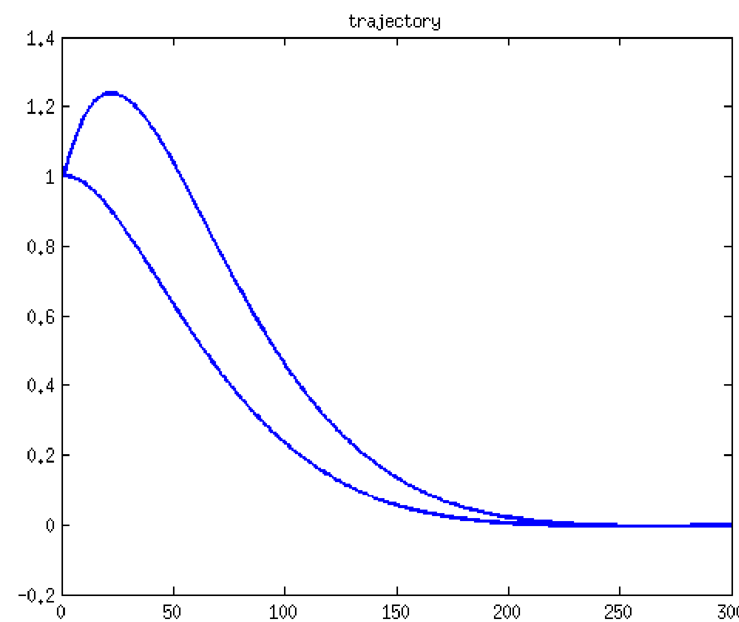
$$B = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ dt & 0 \\ 0 & dt \end{pmatrix}$$

$$u = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

Solve DARE; simulate for 1000 timesteps

$$Q = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad R = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad x_0 = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \end{pmatrix}$$

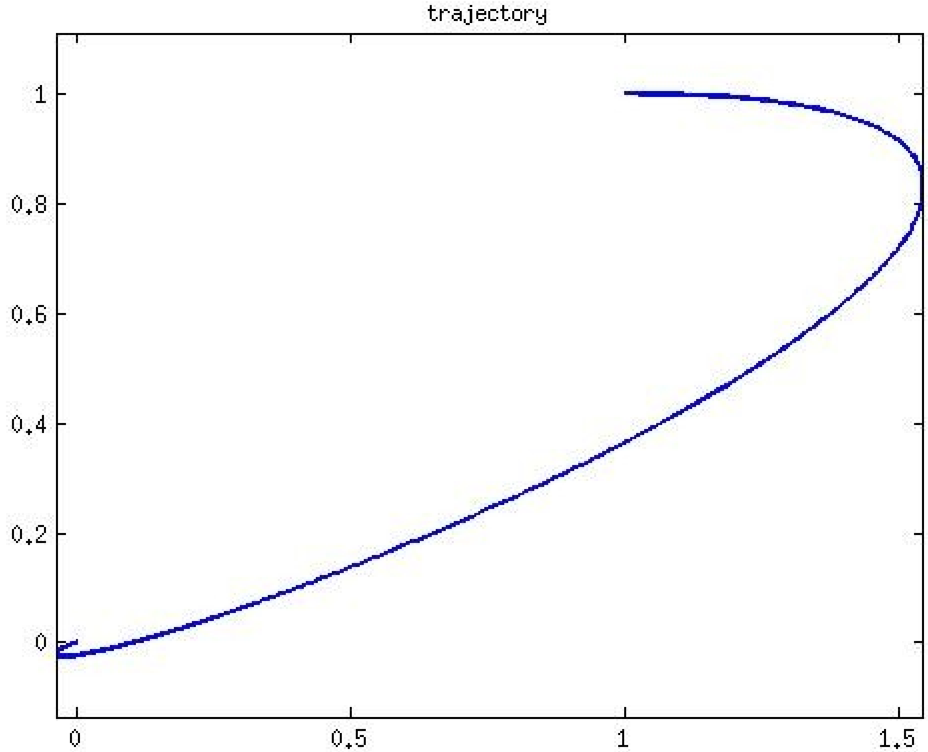




Solve DARE; simulate for 1000 timesteps

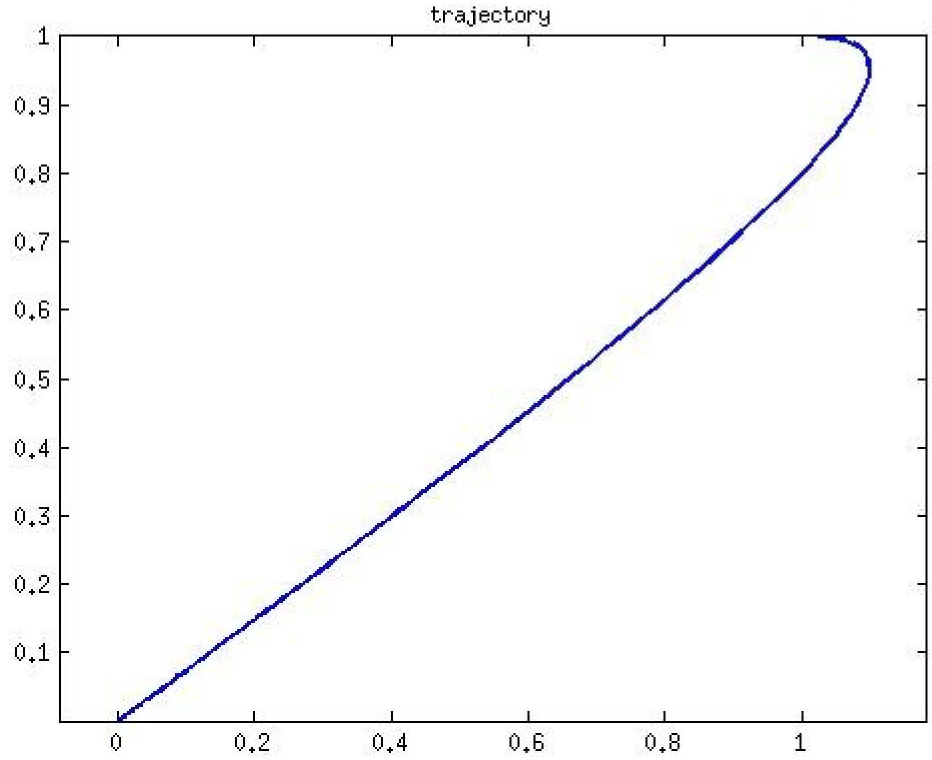
$$Q = \begin{pmatrix} 0.1 & 0 & 0 & 0 \\ 0 & 0.1 & 0 & 0 \\ 0 & 0 & 0.1 & 0 \\ 0 & 0 & 0 & 0.1 \end{pmatrix} R = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$x_0 = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \end{pmatrix}$$



Solve DARE; simulate for 1000 timesteps

$$Q = \begin{pmatrix} 10 & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 \\ 0 & 0 & 10 & 0 \\ 0 & 0 & 0 & 10 \end{pmatrix} \quad R = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad x_0 = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \end{pmatrix}$$



Solution from a bunch of different starting points

$$Q = \begin{pmatrix} 10 & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 \\ 0 & 0 & 10 & 0 \\ 0 & 0 & 0 & 10 \end{pmatrix} \quad R = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$x_0 = \begin{pmatrix} grid \\ grid \\ 1 \\ 0 \end{pmatrix}$$