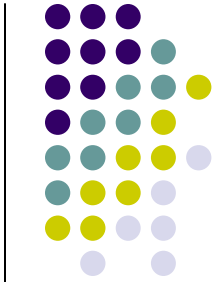
Davis Logemann Loveland Algorithm Framework

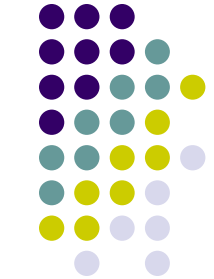


```
while(1) {
   if (decide_next_branch()) { //Branching
       while(deduce()==conflict) { //Deducing
              blevel = analyze_conflicts();
              if (blevel < 0)
                     return UNSAT;
              else back_track(blevel); //Backtracking
       }
   else //no branch means all variables got assigned.
       return SATISFIABLE;
   }
```

Chronological Backtracking

- Backtracking to the highest decision level that has not been tried with both values
- Originally proposed in the DLL paper in 1962
- OK for randomly generated instances, bad for instances generated in practical applications
- We can do better than that

Conflict Driven Learning and Non-Chronological Backtracking



- Marques-Silva and Sakallah [SS96,SS99]

  J. P. Marques-Silva and K. A. Sakallah, "GRASP -- A New Search Algorithm for Satisfiability," Proc. ICCAD 1996.

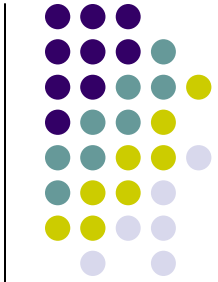
  J. P. Marques-Silva and Karem A. Sakallah, "GRASP: A Search Algorithm for Propositional Satisfiability", *IEEE Trans. Computers*, C-48, 5:506-521, 1999.

- Bayardo and Schrag's RelSAT also proposed conflict driven learning [BS97]

  R. J. Bayardo Jr. and R. C. Schrag "Using CSP look-back techniques to solve real world SAT instances." *Proc. AAAI*, pp. 203-208, 1997

- Practical SAT instances can be solved in reasonable time

Conflict Driven Learning and Non-chronological Backtracking



x1 + x4
x1 + x3' + x8'
x1 + x8 + x12
x2 + x11
x7' + x3' + x9
x7' + x8 + x9'
x7 + x8 + x10'
x7 + x10 + x12'

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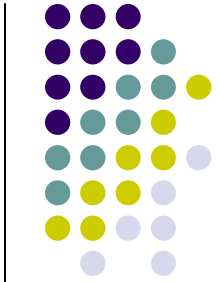
Conflict Driven Learning and Non-chronological Backtracking



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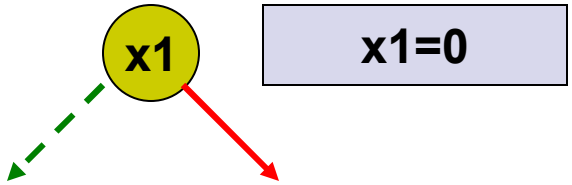
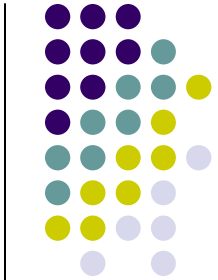


x1=0

Conflict Driven Learning and Non-chronological Backtracking



x1 + x4
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x7' + x8 + x9'
x7 + x8 + x10'
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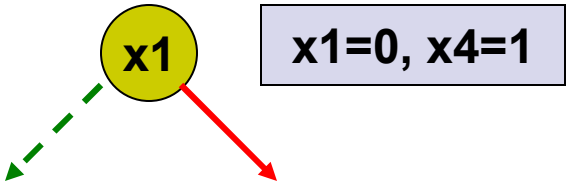

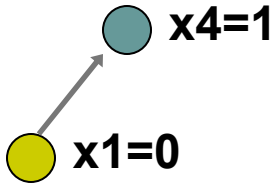

Conflict Driven Learning and Non-chronological Backtracking

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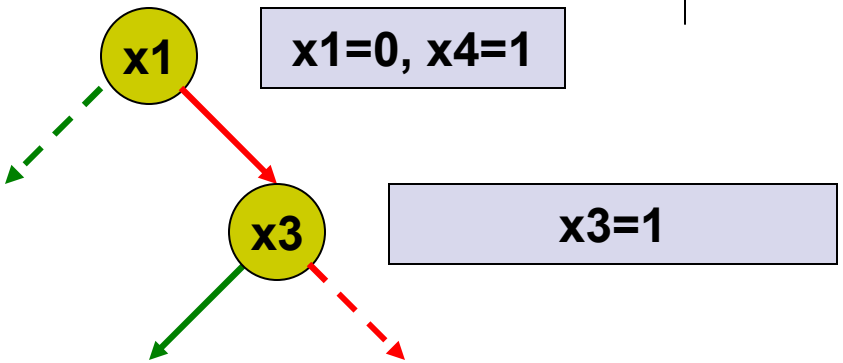

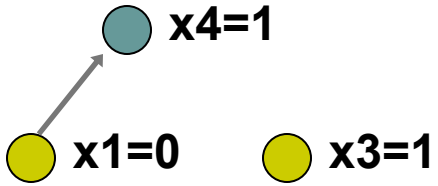

Conflict Driven Learning and Non-chronological Backtracking

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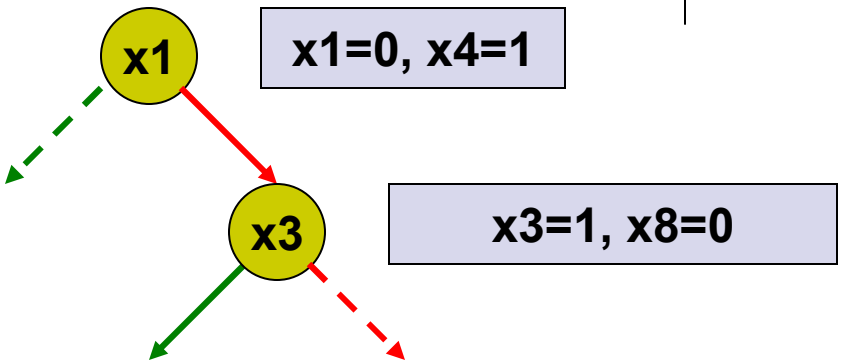

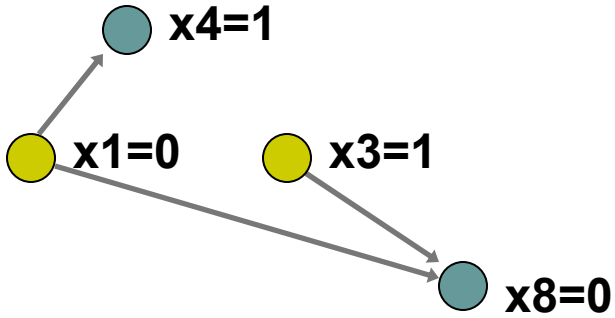

Conflict Driven Learning and Non-chronological Backtracking

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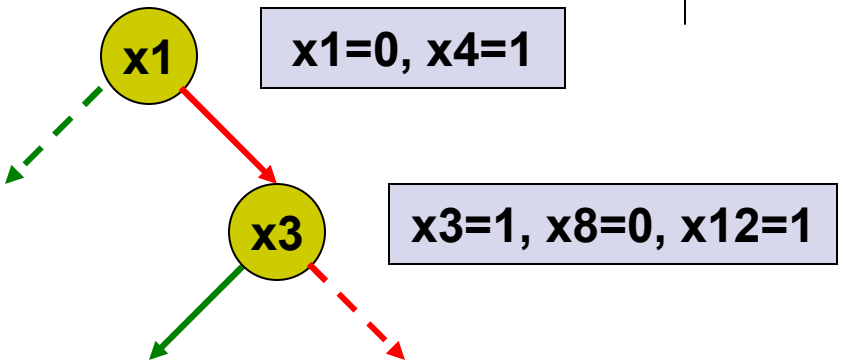

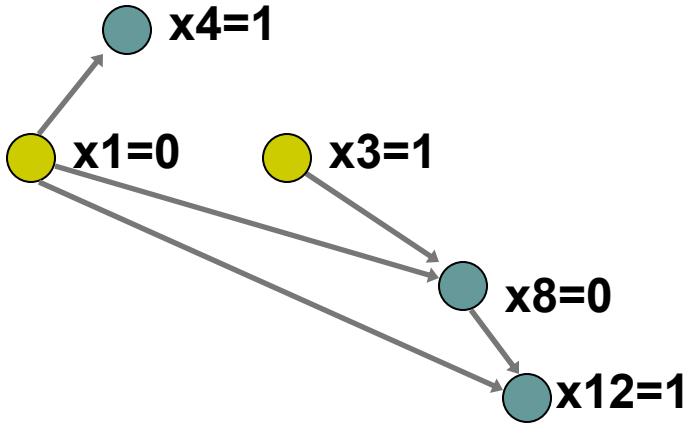

Conflict Driven Learning and Non-chronological Backtracking

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x7 + x8 + x10'
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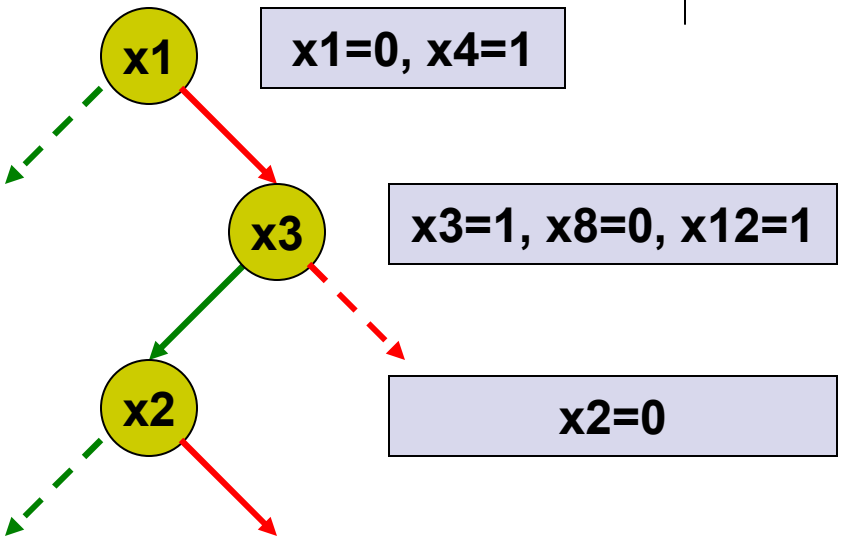

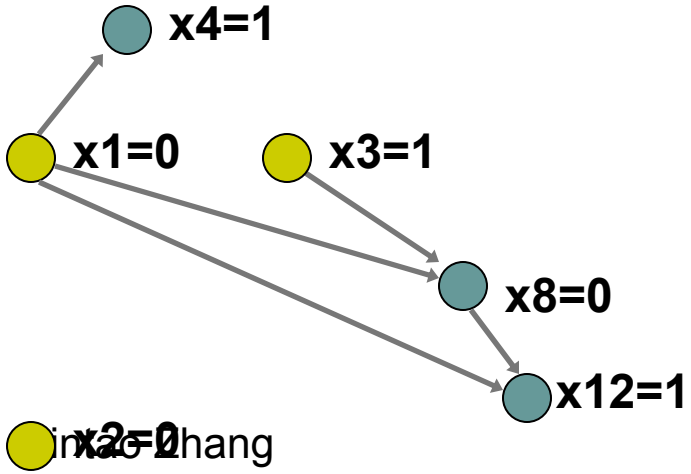

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Conflict Driven Learning and Non-chronological Backtracking

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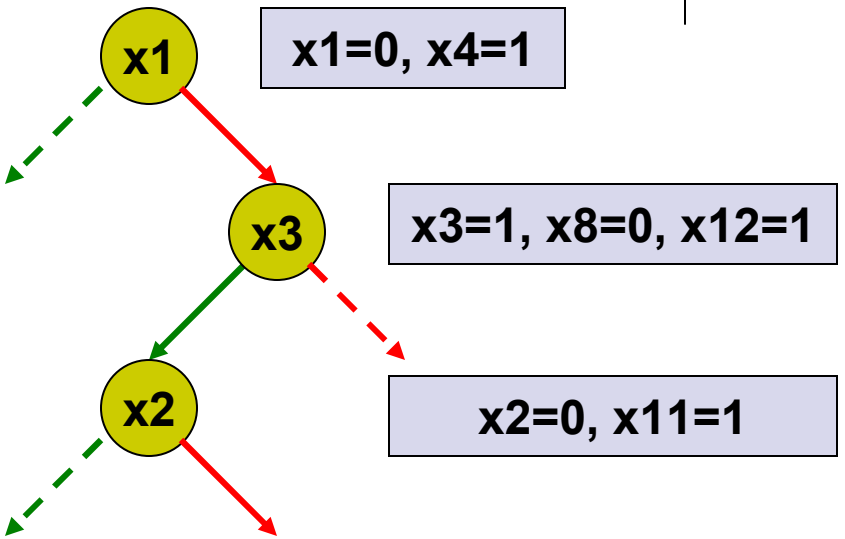

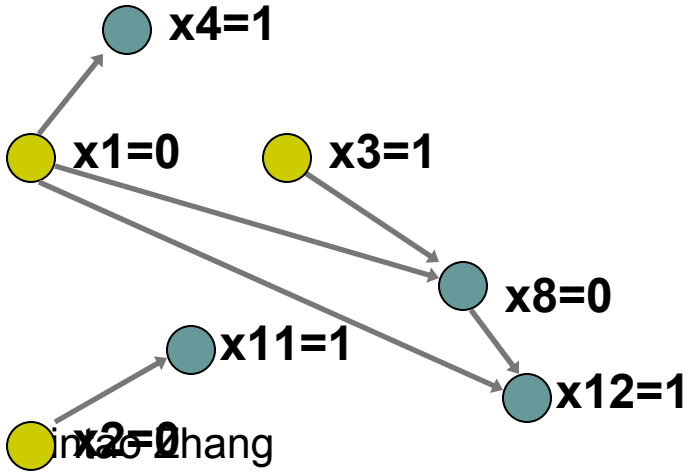

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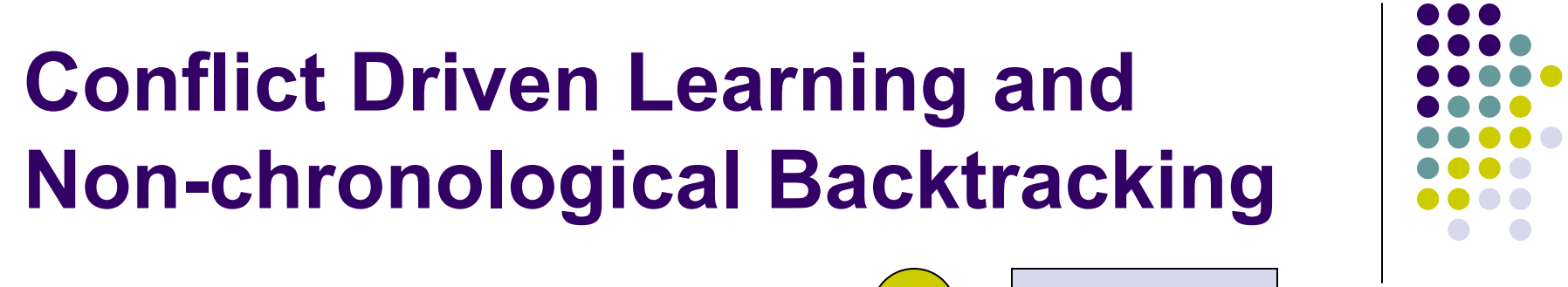
Conflict Driven Learning and Non-chronological Backtracking



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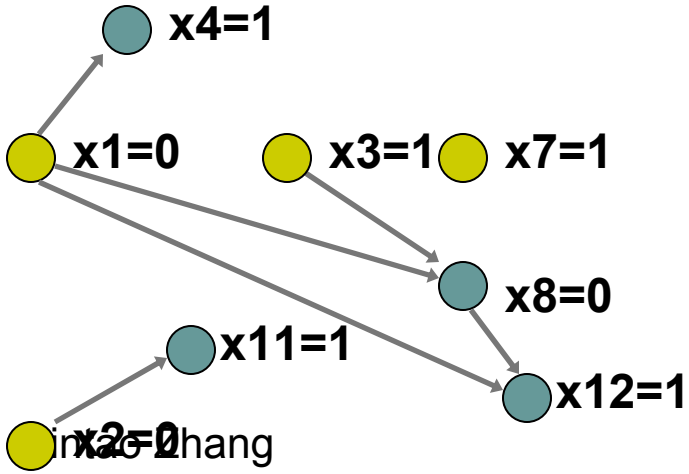

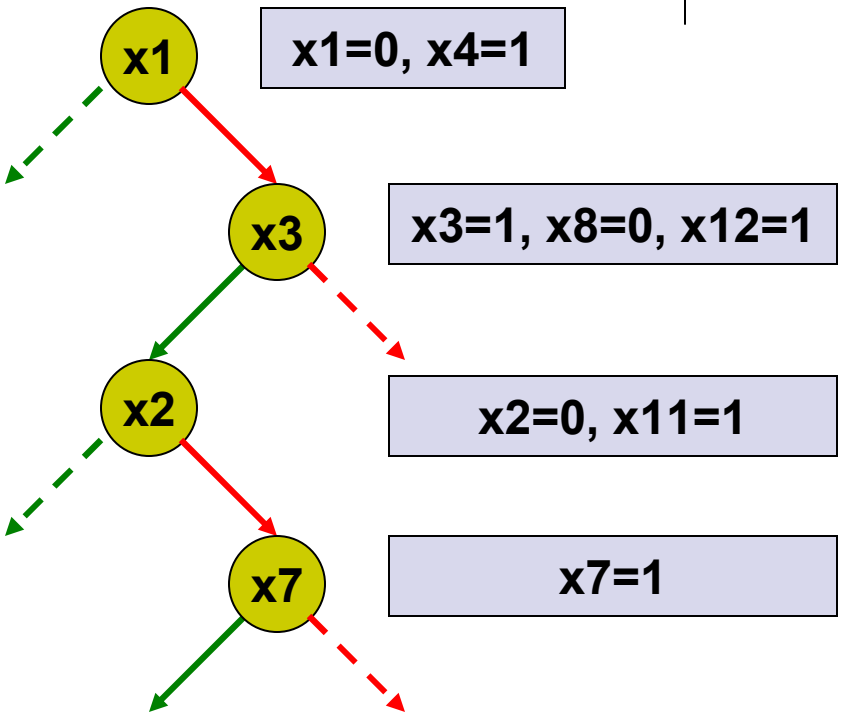

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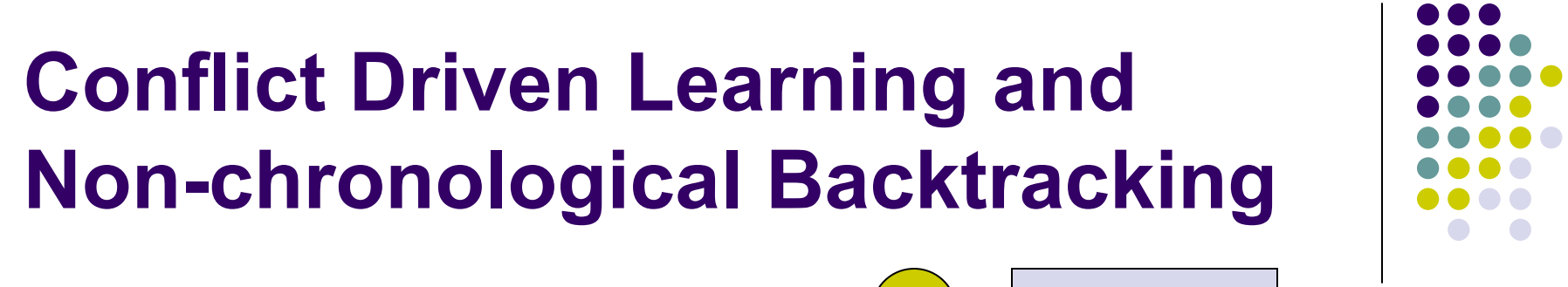
Conflict Driven Learning and Non-chronological Backtracking



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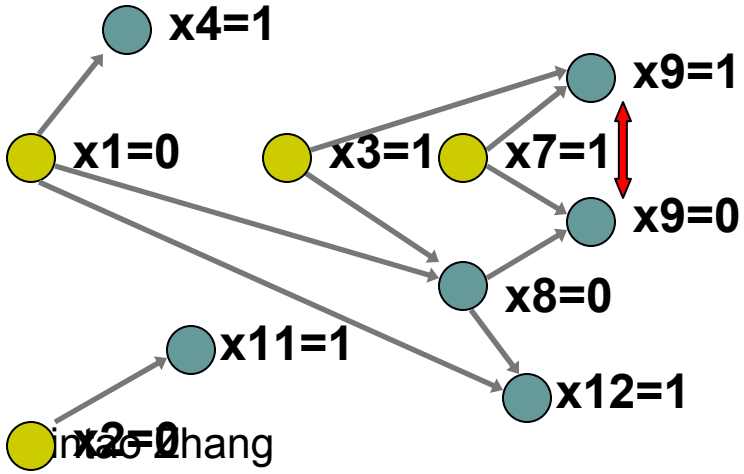

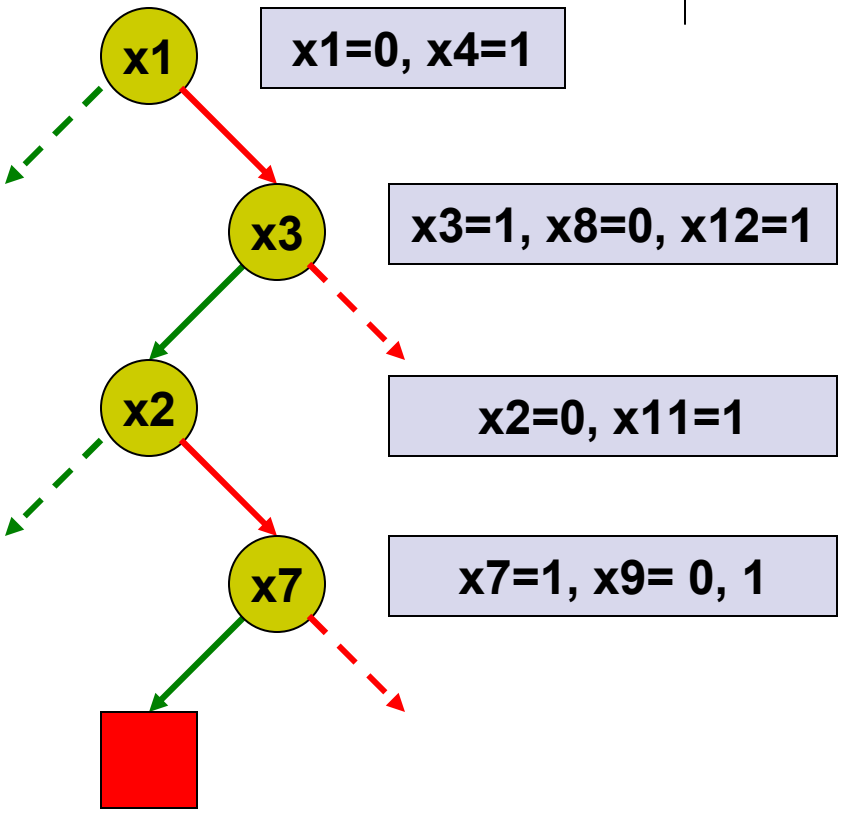

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Conflict Driven Learning and Non-chronological Backtracking

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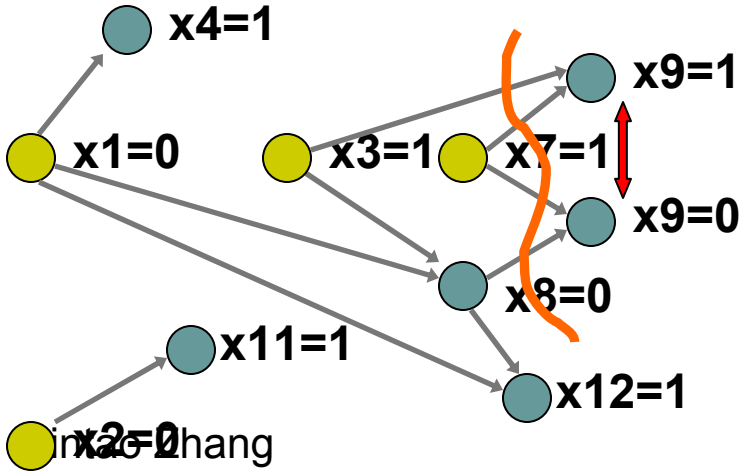

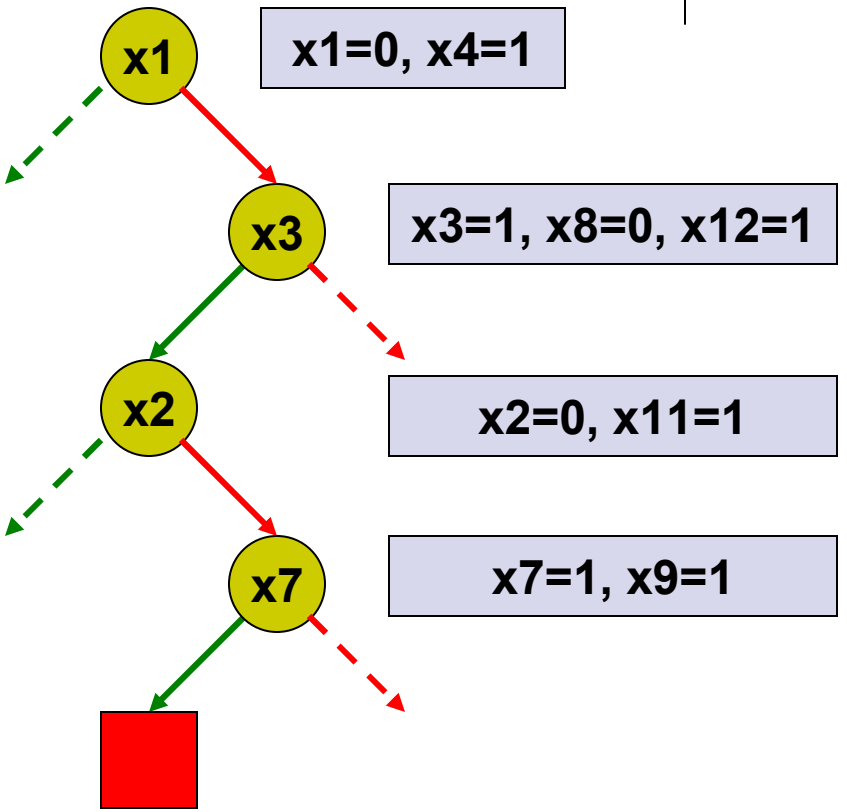

x3=1∧x7=1∧x8=0 → conflict

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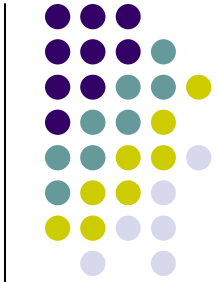
Contra-proposition:



- If a implies b, then b' implies a'

$$x3=1 \wedge x7=1 \wedge x8=0 \rightarrow \text{conflict}$$

$$\text{Not conflict} \rightarrow (x3=1 \wedge x7=1 \wedge x8=0)'$$

$$\text{true} \rightarrow (x3=1 \wedge x7=1 \wedge x8=0)'$$

$$(x3=1 \wedge x7=1 \wedge x8=0)'$$

$$(x3' + x7' + x8)$$

Conflict Driven Learning and Non-chronological Backtracking

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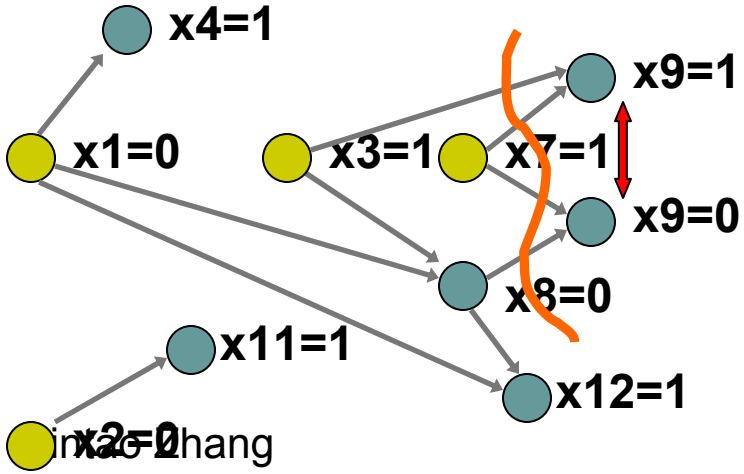
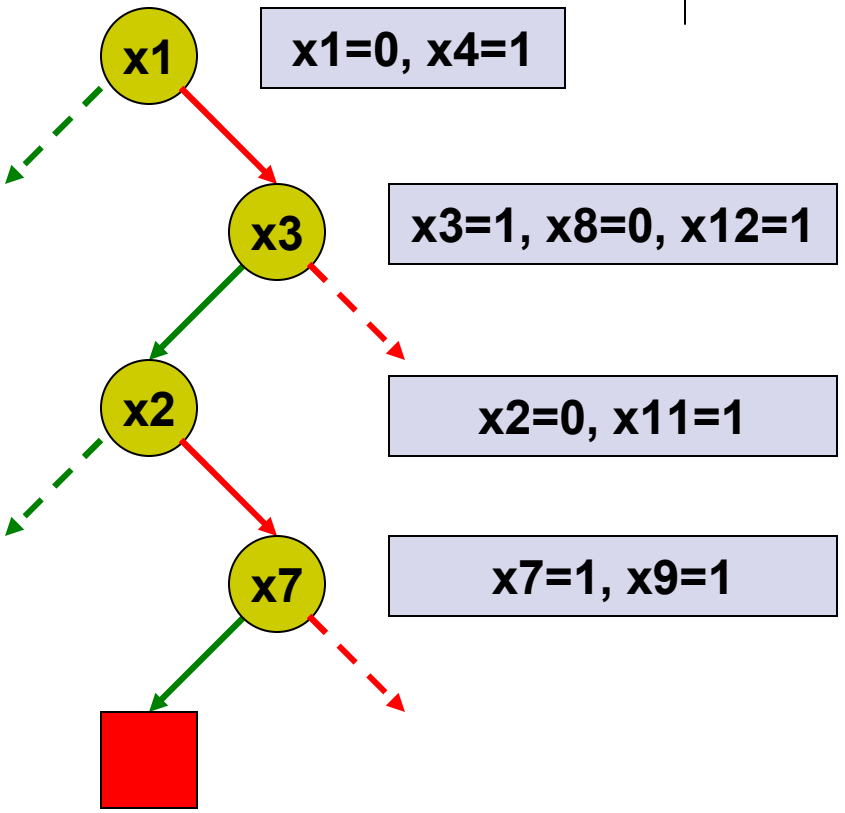




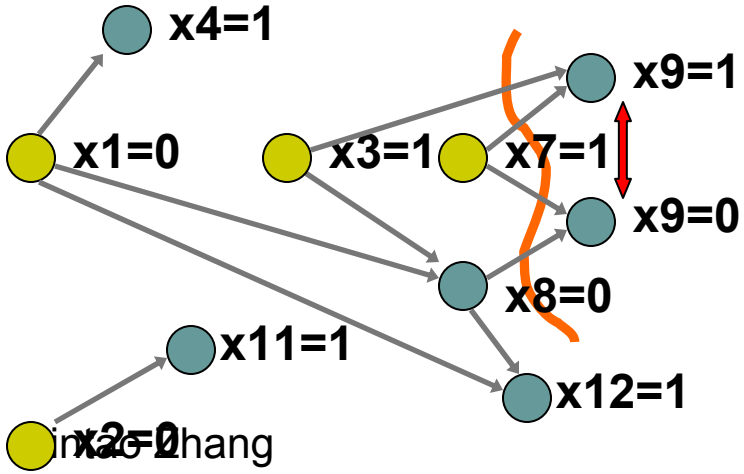
x3=1∧x7=1∧x8=0 → conflict

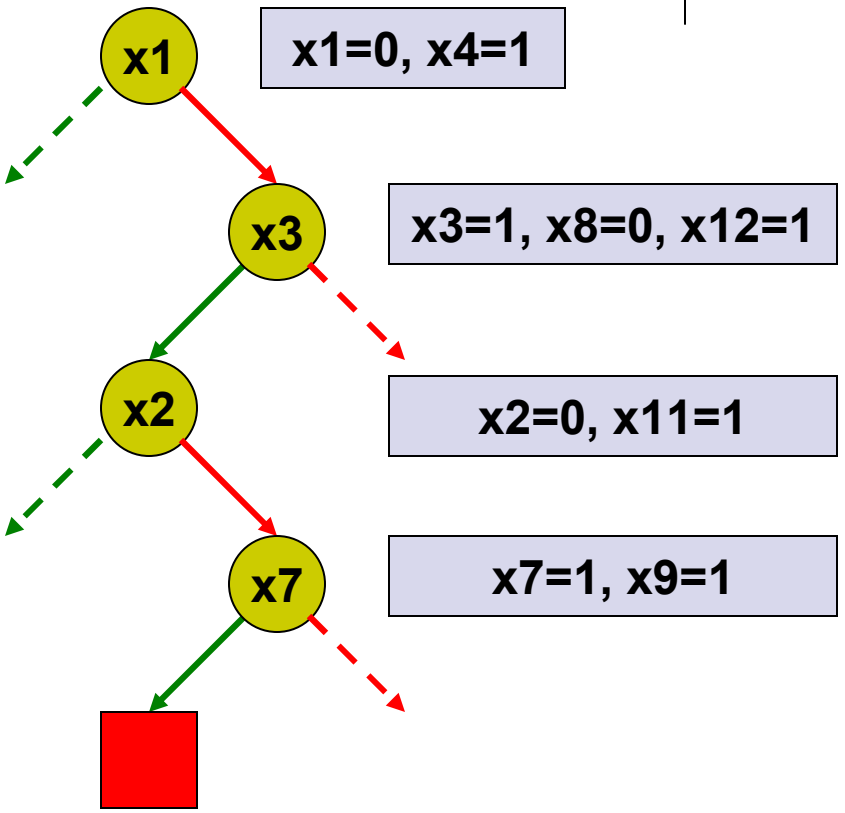
Add conflict clause: x3'+x7'+x8

Conflict Driven Learning and Non-chronological Backtracking

x1 + x4
x1 + x3' + x8'
x1 + x8 + x12
x2 + x11
x7' + x3' + x9
x7' + x8 + x9'
x7 + x8 + x10'
x7 + x10 + x12'

x3'+x7'+x8





x3=1∧x7=1∧x8=0 → conflict

Add conflict clause: x3'+x7'+x8

DLL with Non-Chronological Backtracking and Learning

x1 + x4
x1 + x3' + x8'
x1 + x8 + x12
x2 + x11
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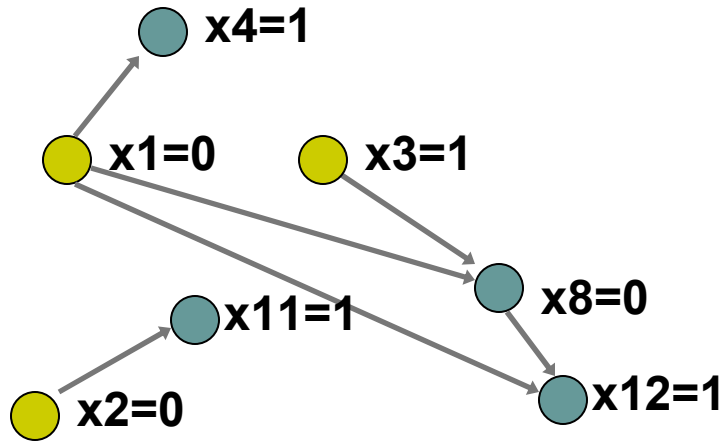
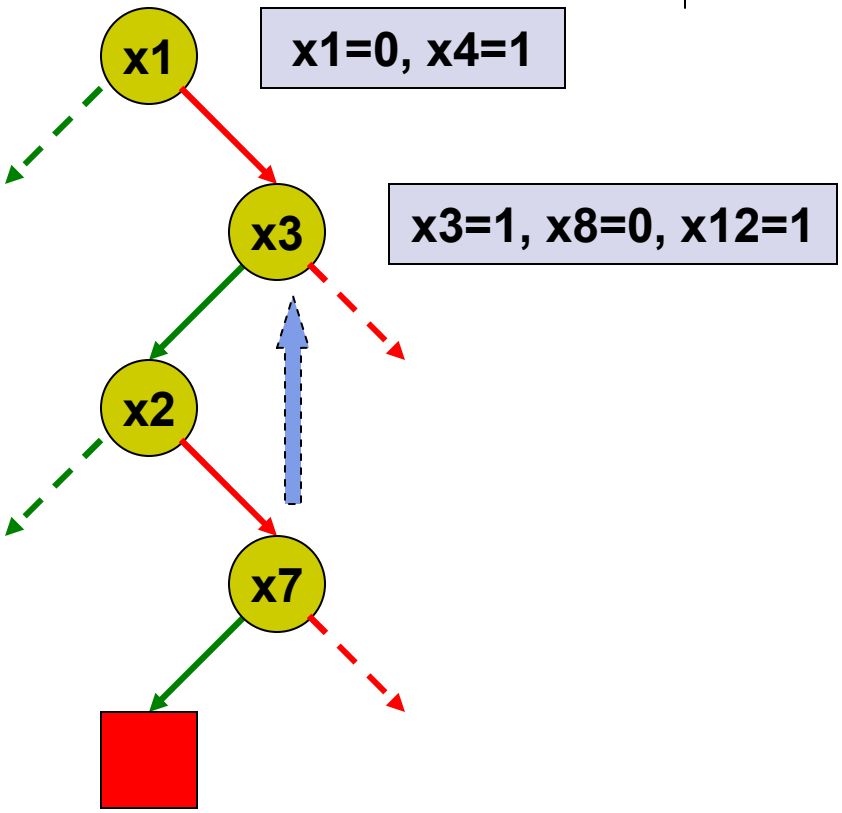
x7' + x8 + x9'
x7 + x8 + x10'
x7 + x10 + x12'
x3' + x8 + x7'





Backtrack to the decision level of x3=1: x7 = 0

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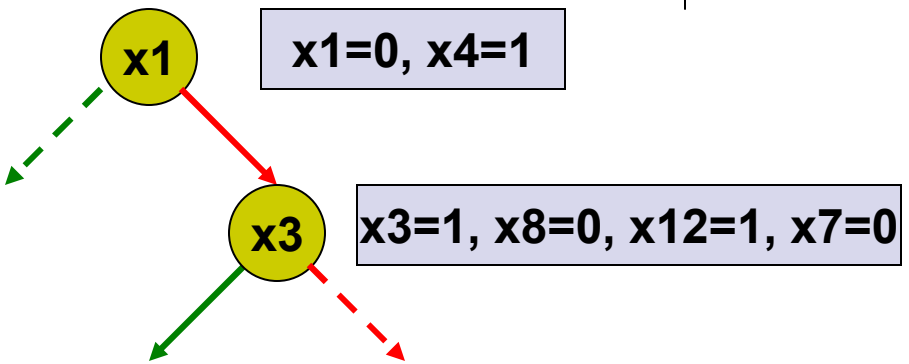
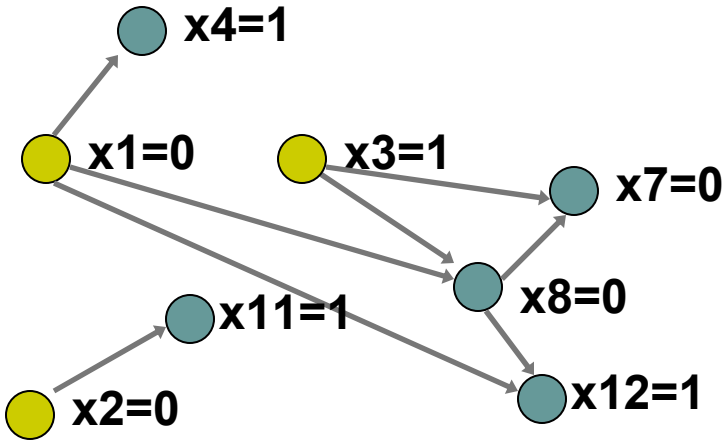
DLL with Non-Chronological Backtracking and Learning

x1 + x4
x1 + x3' + x8'
x1 + x8 + x12
x2 + x11
x7' + x3' + x9
x7' + x8 + x9'
x7 + x8 + x10'
x7 + x10 + x12'
x3' + x8 + x7'





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