HWK3 Decision Procedures
Due 10/31/2007 *before class starts* Panagiotis Manolios

Exercise 1. Unambiguously describe the BDD for the following class of Boolean constraints: exactly k of $x_1, x_2, \ldots, x_n$ are true, for $0 < k < n$. How many BDD nodes are there (as a function of k and n)? What is the most concise equisatisfiable CNF formula you can define?

Exercise 2. Temporal Logic.

1. Simplify the following CTL* formulas as much as possible: (a) $\mathsf{AE}\neg(true\mathsf{U}g)$, (b) $\mathsf{A}[f\mathsf{U}(f\mathsf{U}g)]$. You can (and should) use any of the temporal operators we discussed in class (*i.e.*, even the ones that are technically abbreviations, such as G).
2. Give mu-calculus characterizations of $\mathsf{AF}p$ and $\mathsf{E}(f\mathsf{U}g)$ that do not use μ (use ν instead).

Exercise 3. Prove the following theorem:
Theorem Let f be a monotonic function on $\langle L, \vee, \wedge, \leq\rangle$, a complete lattice. Let $S = \{b \mid b \leq f.b\}, \alpha = \vee S$. Then α is the greatest fixpoint of f.

Exercise 4. Let $f : \mathcal{P}(S) \rightarrow \mathcal{P}(S)$ such that $a \subseteq b \quad \Rightarrow \quad f.a \subseteq f.b$ for all $a, b \in S$. By the Tarski-Knaster fixpoint theorem, $\mu.f = \langle \cup \alpha \in On :: f^{\alpha}(\emptyset)\rangle$, where On is the class of ordinals. What is relevant here is that we need to iterate past the natural numbers. Give an example of a function satisfying the above constraints, where S is $\mathbb{N}$ and where iterating over all the natural numbers does not result in a fixpoint. (Make sure the f you provide is monotonic. Also, $f^{\omega}(\emptyset) = \cup_{i \in \omega} f^{i}(\emptyset)$, so you just have to show that $f^{\omega}(\emptyset)$ is not a fixpoint.)

Exercise 5. Describe Büchi automata that accept the languages defined by the following LTL formulas. Assume that $\Sigma = \{a, b, c\}$. (a) $(\mathsf{F}a)\mathsf{U}(\mathsf{G}b)$ and (b) $(\mathsf{GF}a) \;\Rightarrow\; (\mathsf{GF}b)$.